Design For Assembly (DFA)

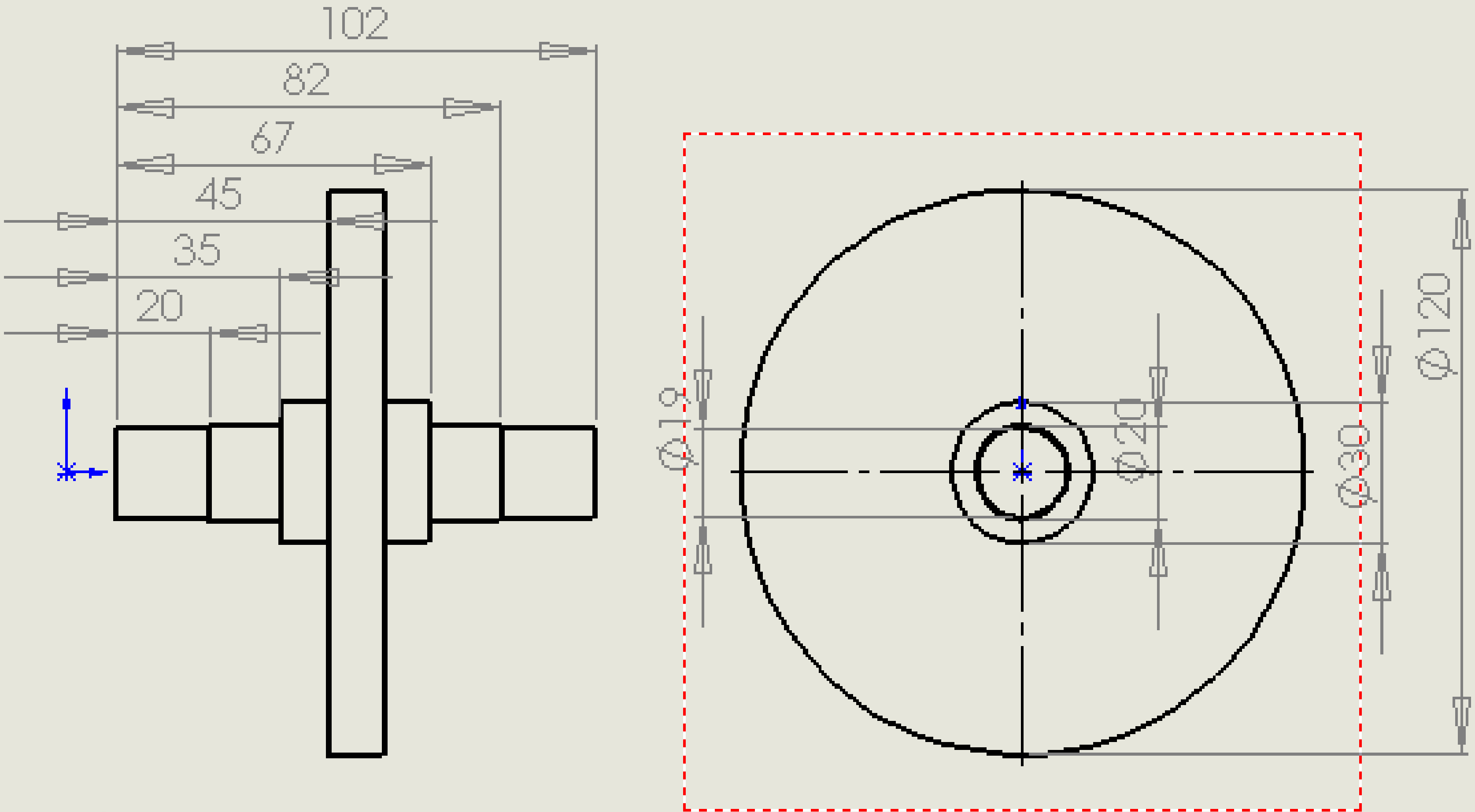
1. Utilize Common Parts and Materials (standardization)

- To facilitate **design activities**, to minimize the amount of **inventory** in the system, and to standardize **handling and assembly operations**.
- Limit exotic or unique components because suppliers are less likely to compete on quality or cost for these components.

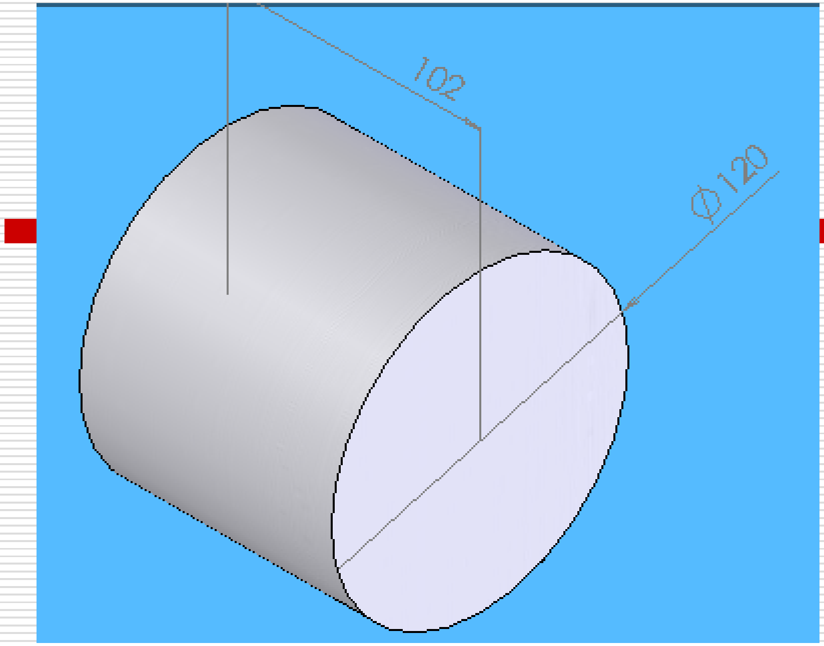
Product catalogue

Supplier list

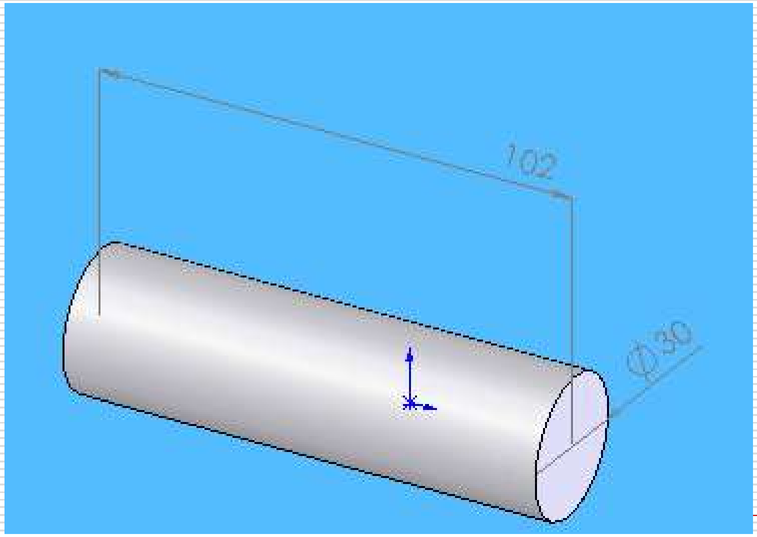
2. Minimize Material Cost



Weight = 1.33 Kg

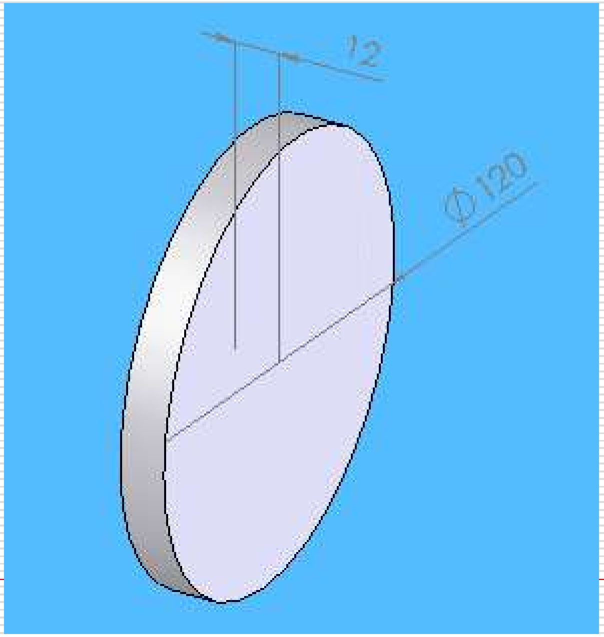


Weight = 9 Kg



Weight = 0.563 Kg

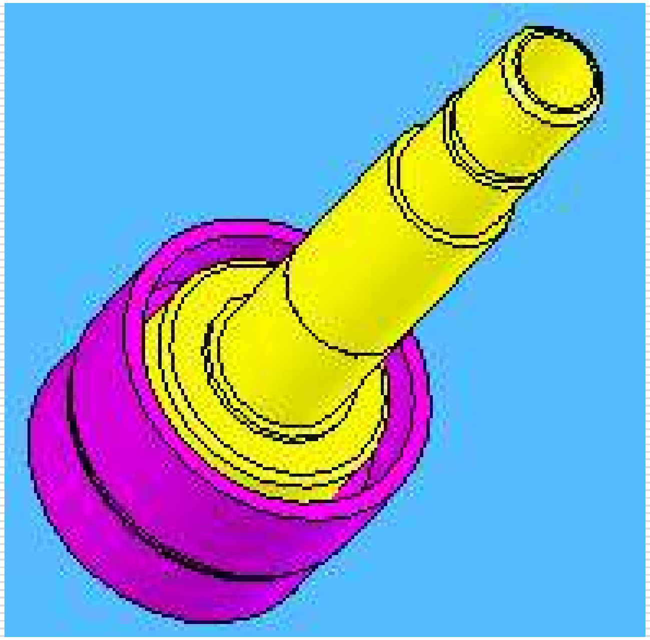
+

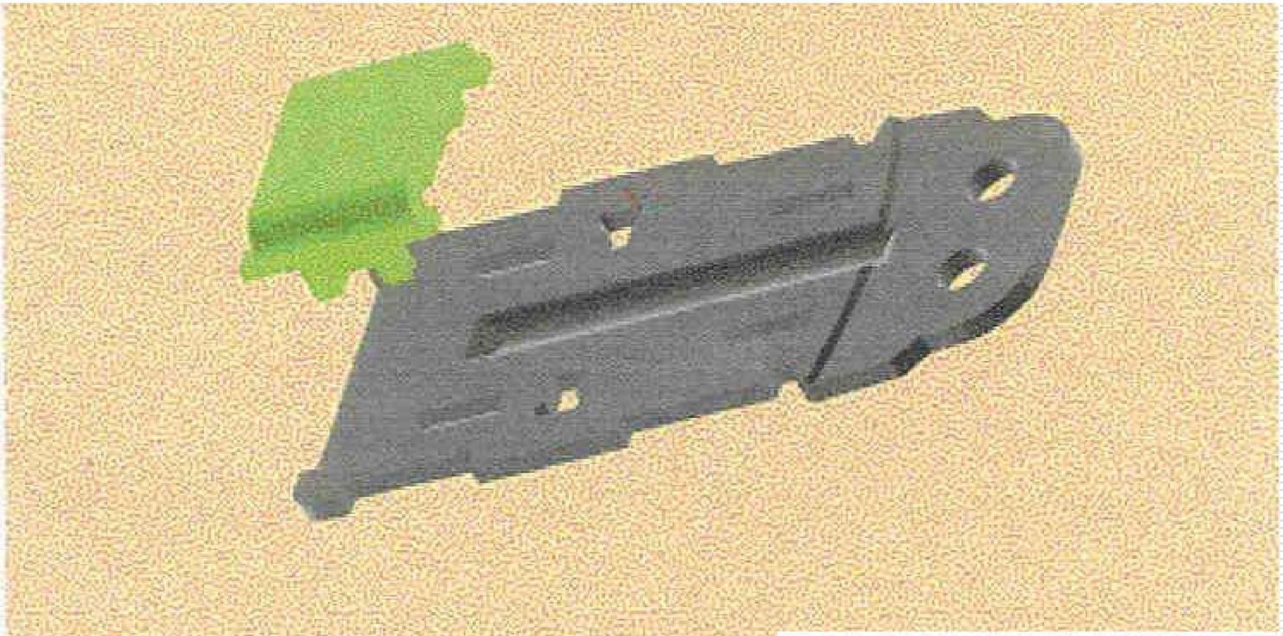


Weight = 1.05 Kg

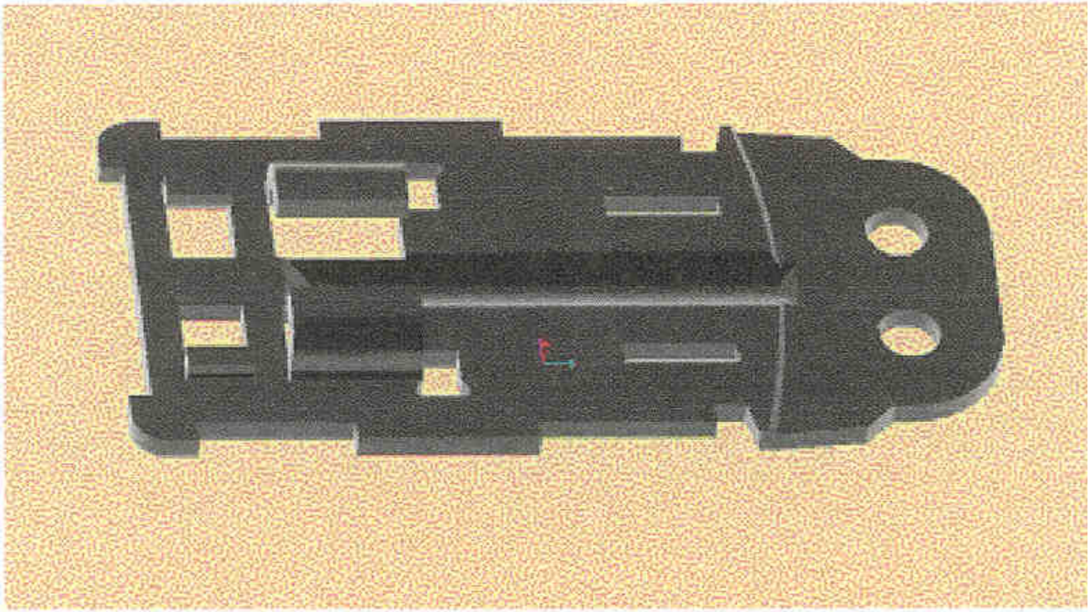
3. Design for minimum number of Parts

- To determine the theoretical minimum number of parts, ask the following:
  - Does the part move relative to all other moving parts?
  - Must the part absolutely be of a different material from the other parts?
  - Must the part be different to allow possible disassembly?



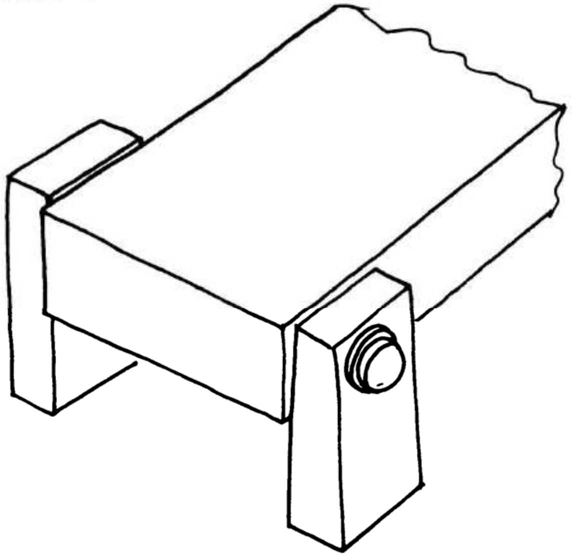


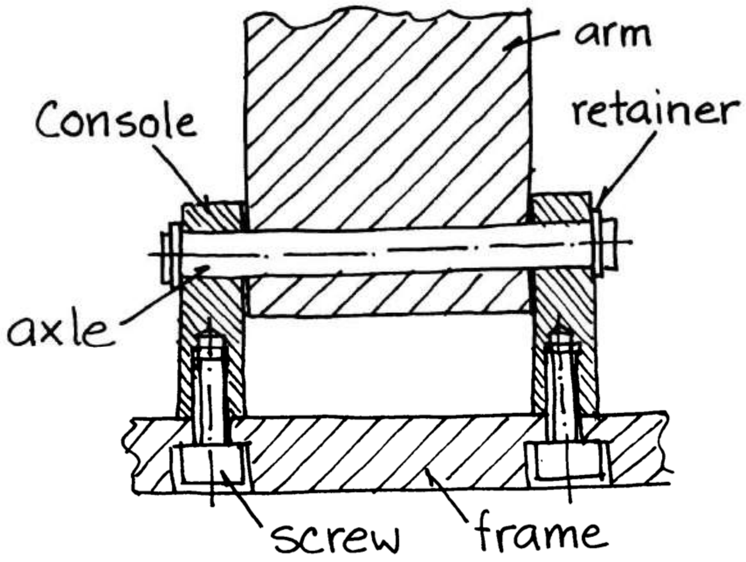
Bridge and Base-plate i



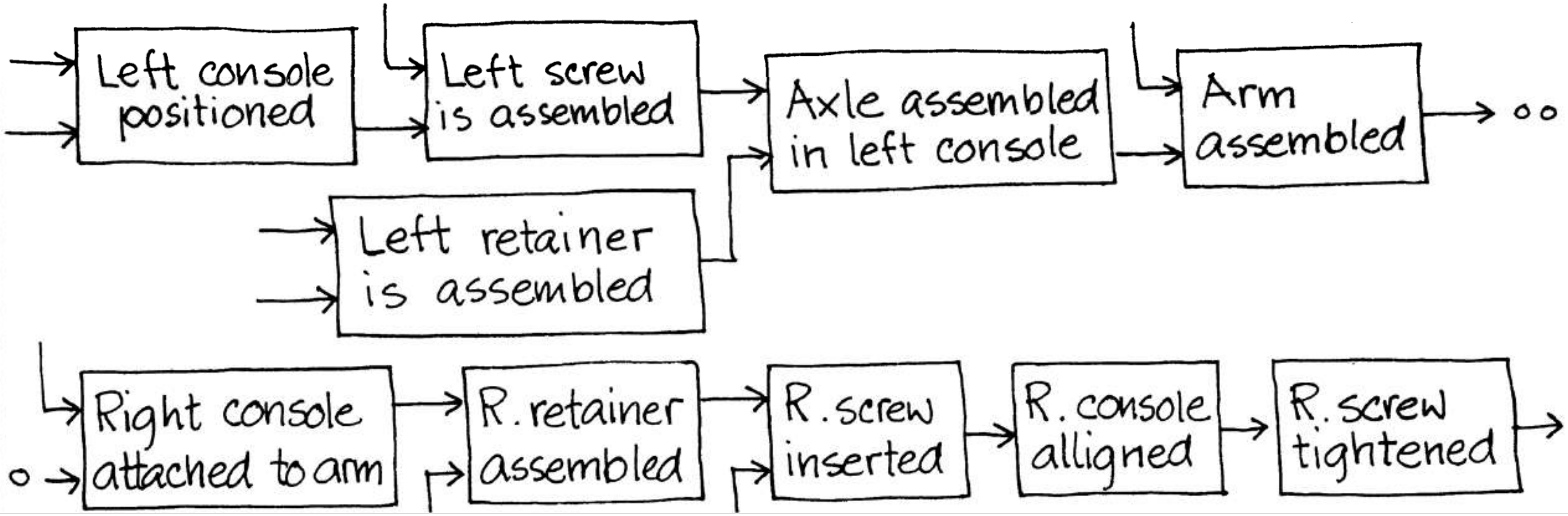
Recommendation for single part that performs bridge and base-plate functions

Alternative 1:

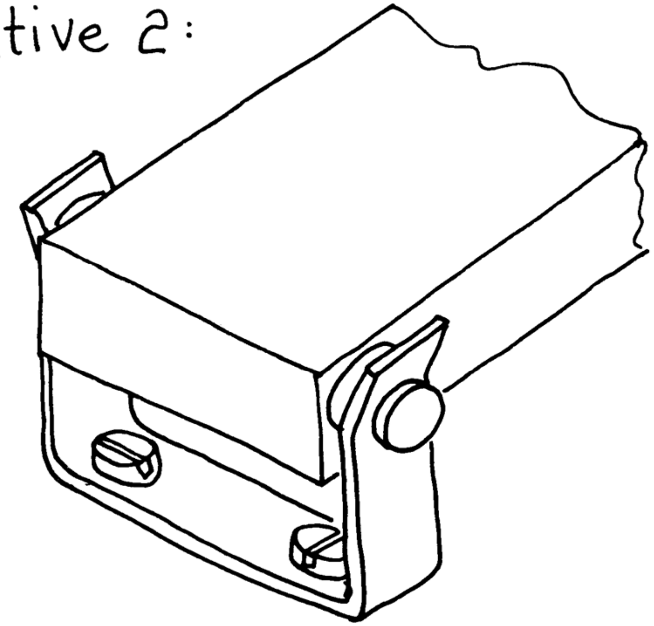


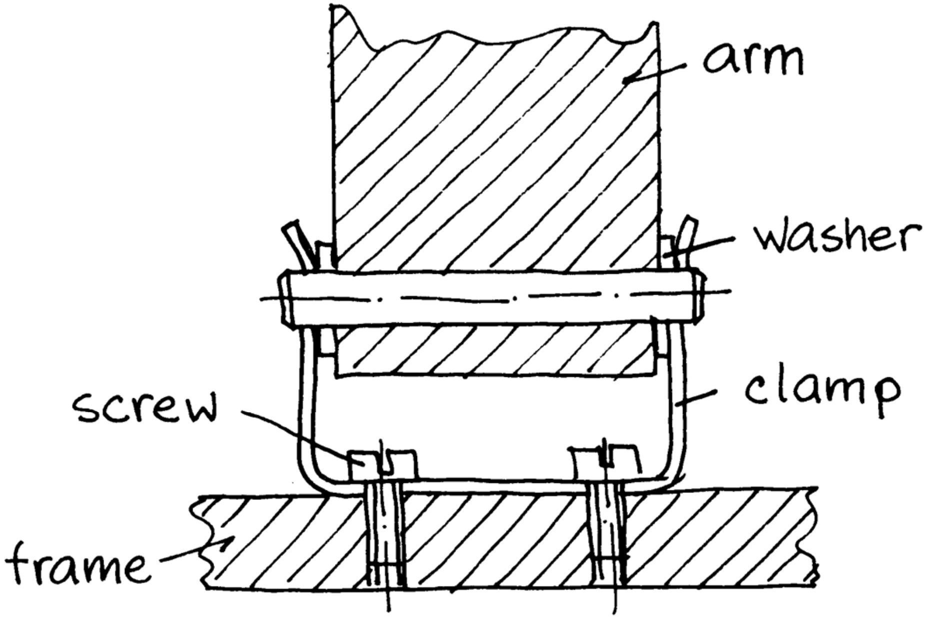


Assembly process:

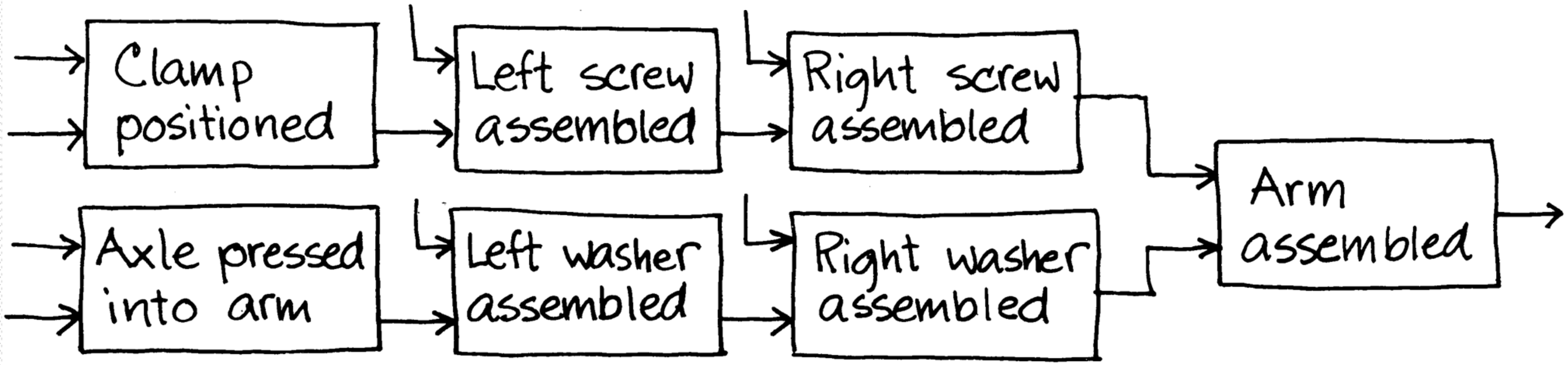


Alternative 2:





Assembly process:



6. Reduce Assembly Time

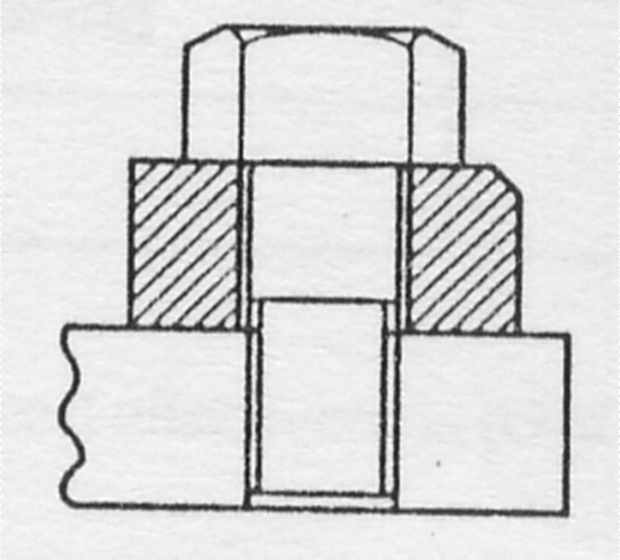
How is part acquired, oriented, made ready for insertion?

How is it inserted/ fastened?

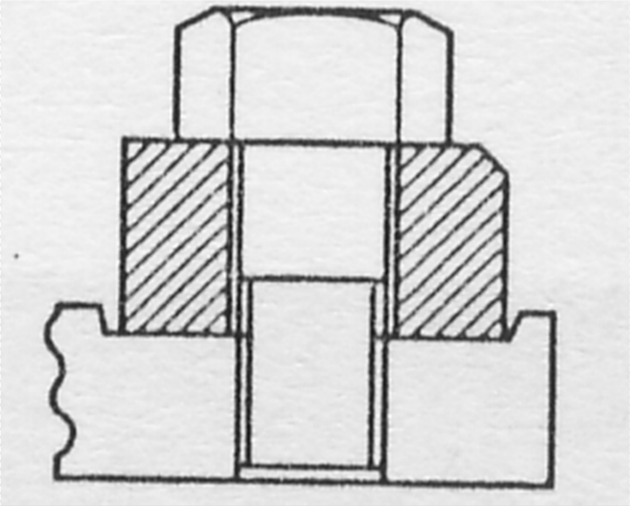
Reference: Boothroyd, Dewhurst, Winston, “Product Design for Manufacture and Assembly”, 1994, In IIT Library

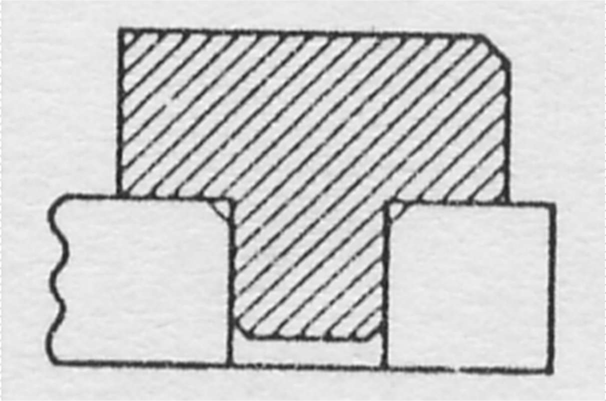
	Easy to align and insert	Not easy to align or insert	Not easy to align and insert	Severe difficulties
No access or vision difficulties	1.5	3.0	4.5	7.5
Obstructed access or restricted vision	3.7	5.2	6.7	9.7
Obstructed access and restricted vision	5.9	7.4	8.9	11.9

CASE I



	Easy to align and insert	Not easy to align or insert	Not easy to align and insert	Severe difficulties
No access or vision difficulties	1.5	3.0	4.5	7.5
Obstructed access or restricted vision	3.7	5.2	6.7	9.7
Obstructed access and restricted vision	5.9	7.4	8.9	11.9





CASE II

Designing motor-drive assembly to sense & control its position on two steel guide rails.

Motor must be fully enclosed for aesthetic reasons and have removable cover for access so that position sensor can be adjusted.

Motor and sensor have wires that connect them to a power supply and a control unit, respectively.

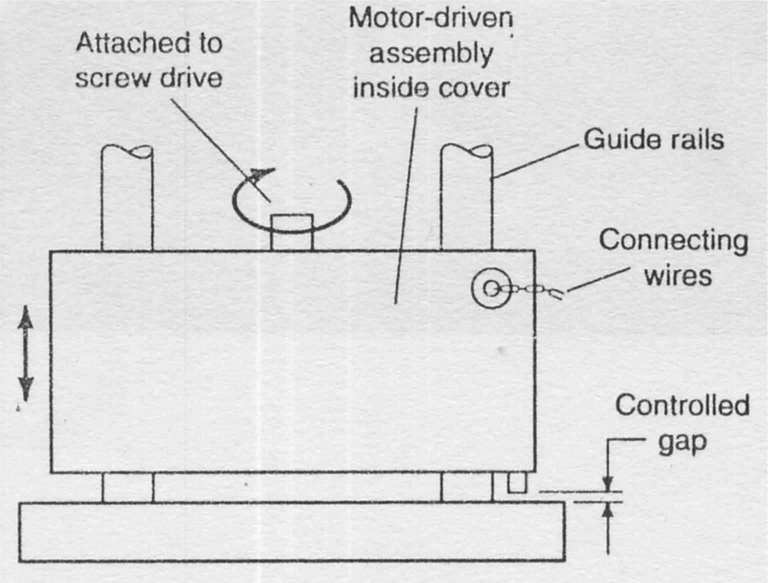
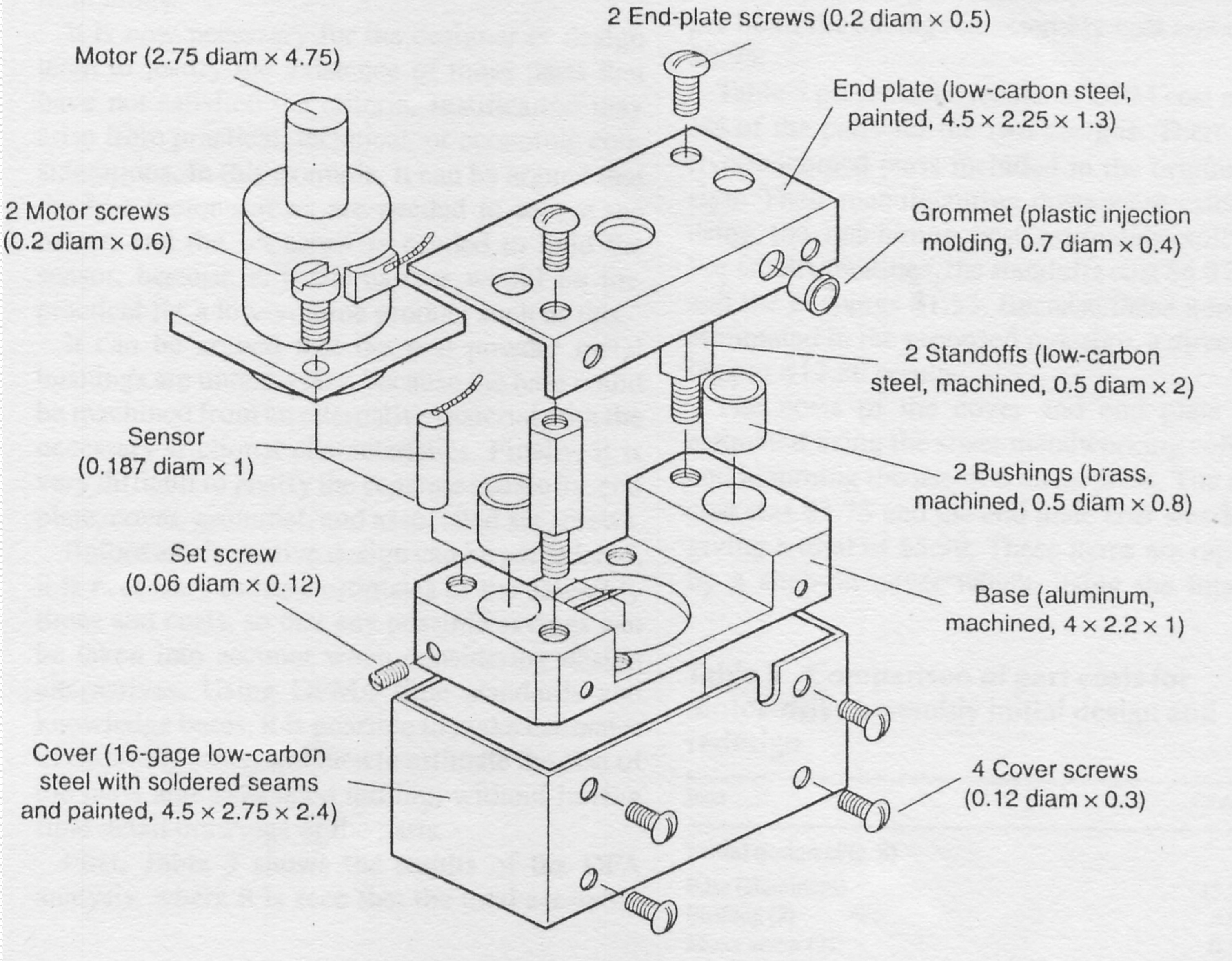
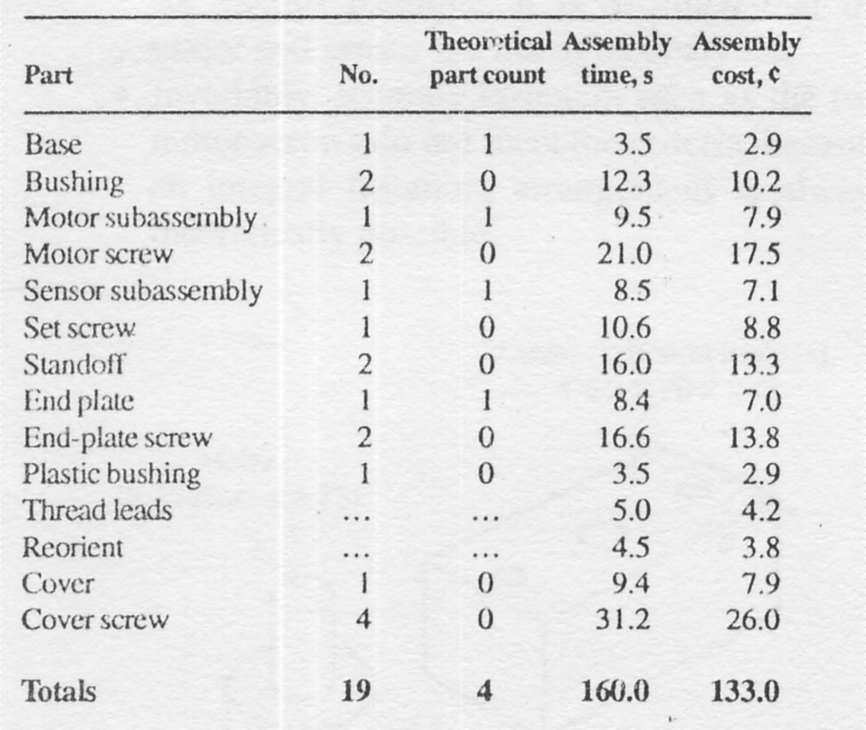


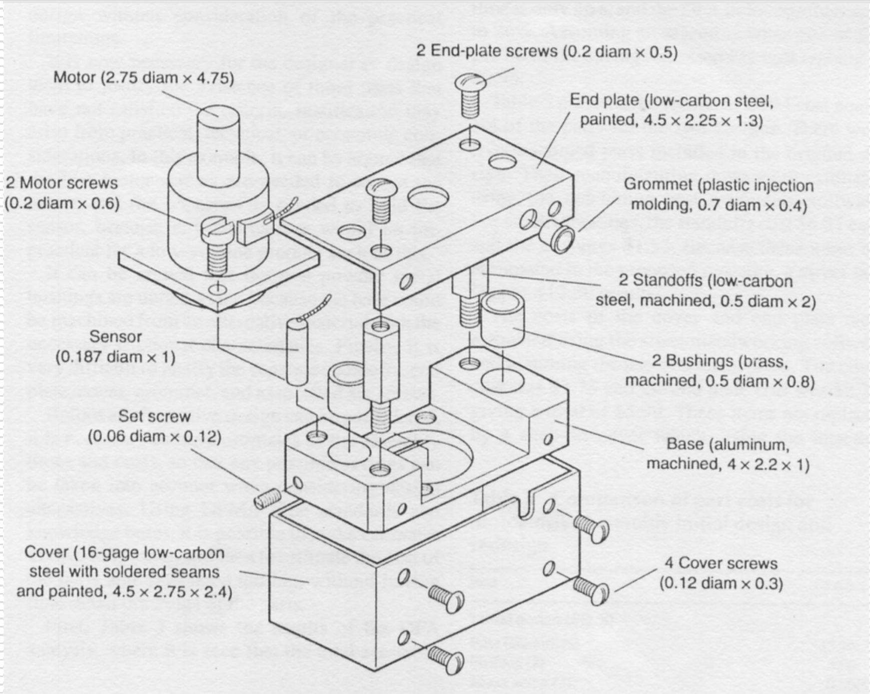
Fig: Motor Drive Assembly

Fig. Initial design of motor drive assembly





Part	No.	Theoretical part count	Assembly time, s	Assembly cost, ¢
Base	1	1	3.5	2.9
Bushing	2	0	12.3	10.2
Motor subassembly	1	1	9.5	7.9
Motor screw	2	0	21.0	17.5
Sensor subassembly	1	1	8.5	7.1
Set screw	1	0	10.6	8.8
Standoff	2	0	16.0	13.3
End plate	1	1	8.4	7.0
End-plate screw	2	0	16.6	13.8
Plastic bushing	1	0	3.5	2.9
Thread leads	...	...	5.0	4.2
Reorient	...	...	4.5	3.8
Cover	1	0	9.4	7.9
Cover screw	4	0	31.2	26.0
Totals	19	4	160.0	133.0



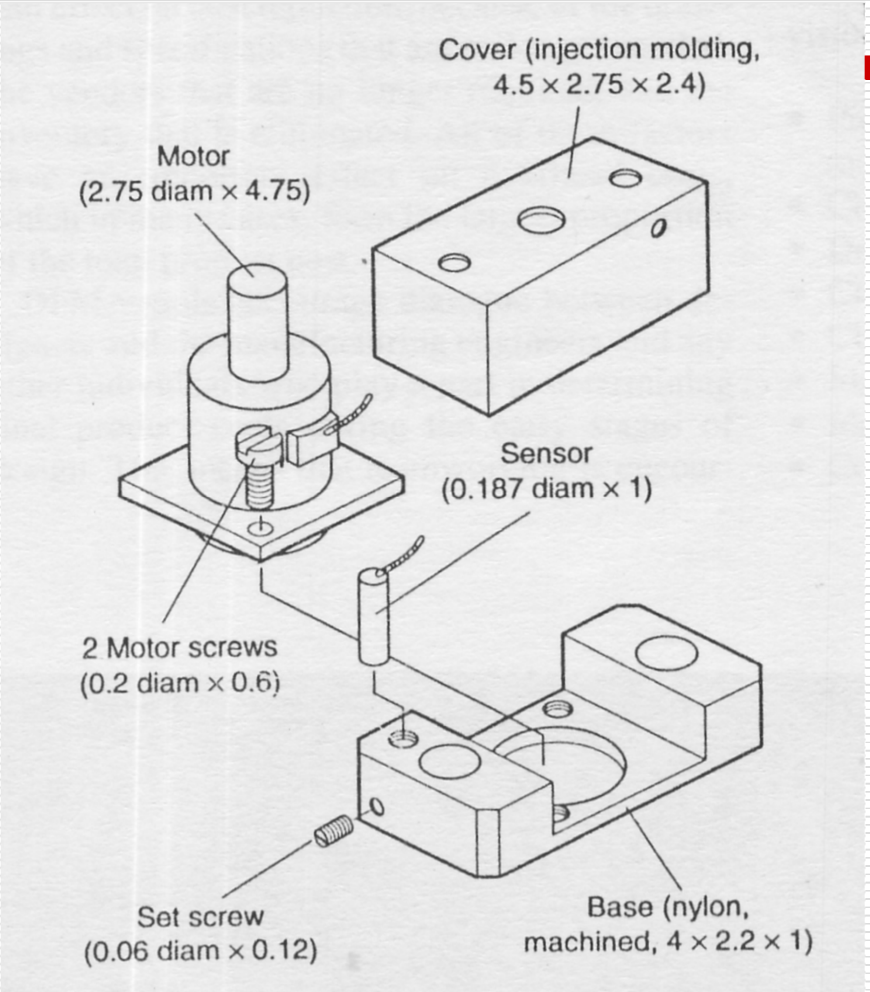
Design for Assembly Index

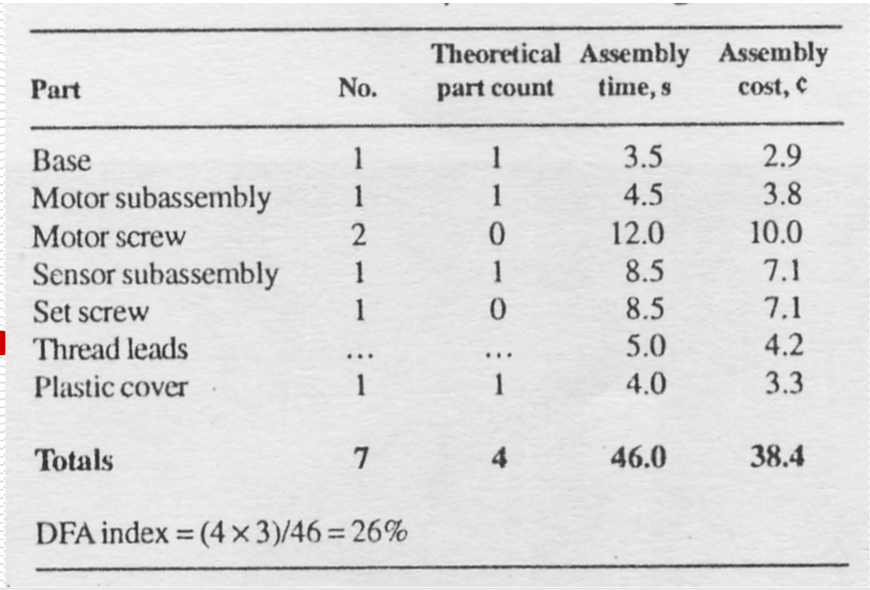
DFA = (theoretical parts * 3)/(Real assembly time)

DFA = 7.5% !!!!!!!!

	Easy to align and insert	Not easy to align or insert	Not easy to align and insert	Severe difficulties
No access or vision difficulties	1.5	3.0	4.5	7.5
Obstructed access or restricted vision	3.7	5.2	6.7	9.7
Obstructed access and restricted vision	5.9	7.4	8.9	11.9

Improved design



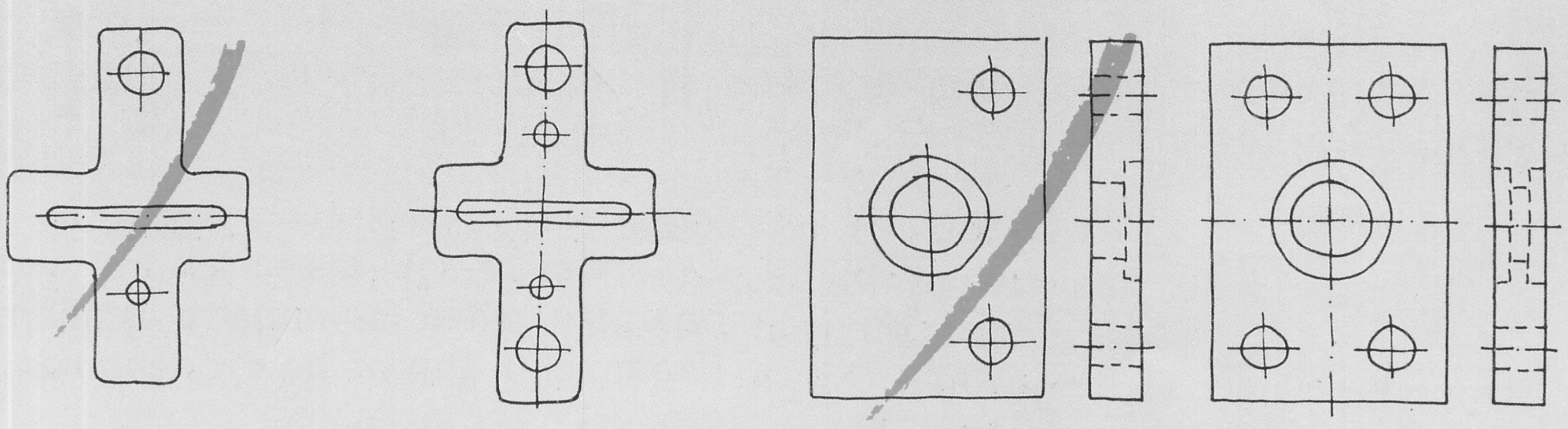


Part	No.	Theoretical part count	Assembly time, s	Assembly cost, ¢
Base	1	1	3.5	2.9
Motor subassembly	1	1	4.5	3.8
Motor screw	2	0	12.0	10.0
Sensor subassembly	1	1	8.5	7.1
Set screw	1	0	8.5	7.1
Thread leads	…	…	5.0	4.2
Plastic cover	1	1	4.0	3.3
Totals	**7**	**4**	**46.0**	**38.4**

DFA index = (4 × 3)/46 = 26%

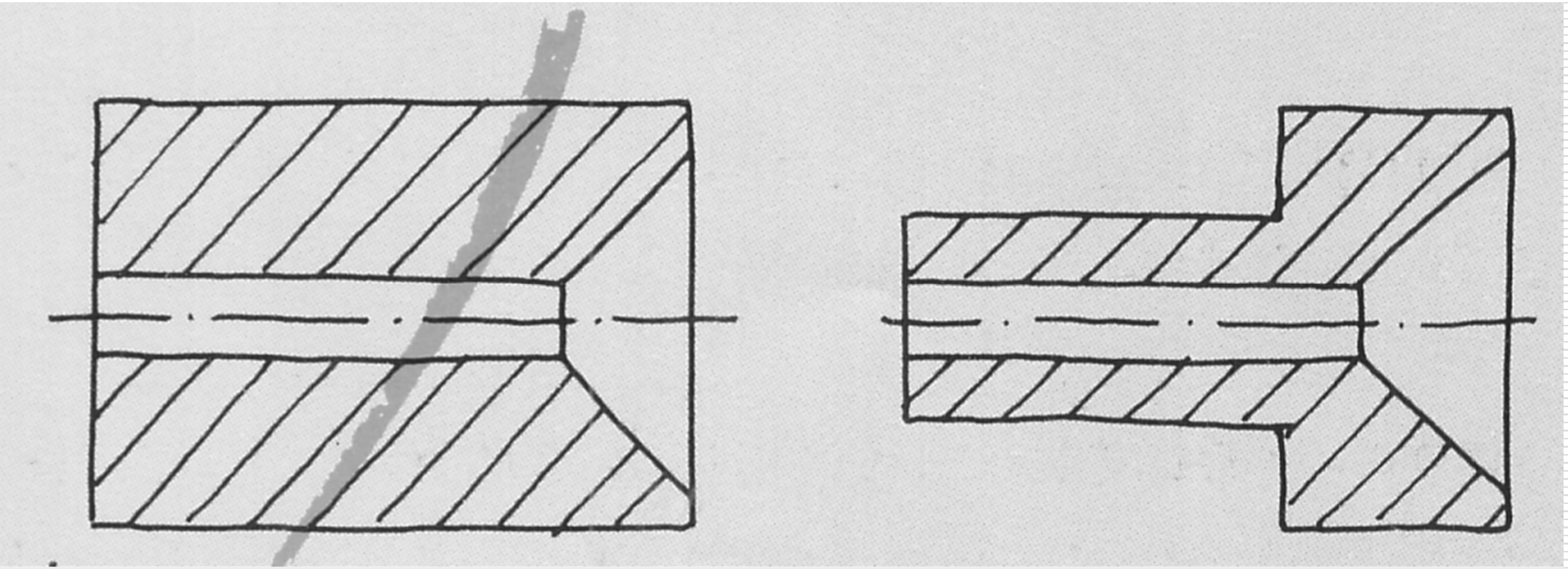
4. Design for Parts Orientation and Handling

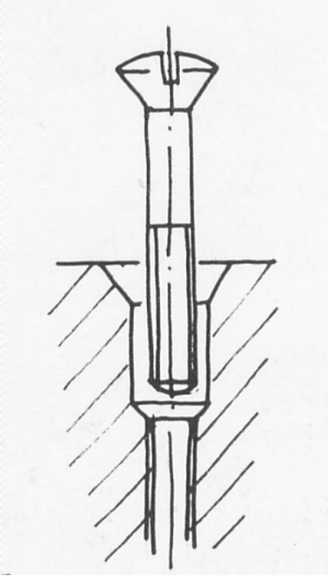
- The less an assembler has to move and orient both the original part and parts to be added, the faster and more trouble-free the process.
  - Part design should incorporate symmetry around both axes of insertion wherever possible.
  - Where parts cannot be symmetrical, the asymmetry should be emphasized to assure correct insertion or easily identifiable feature should be provided.



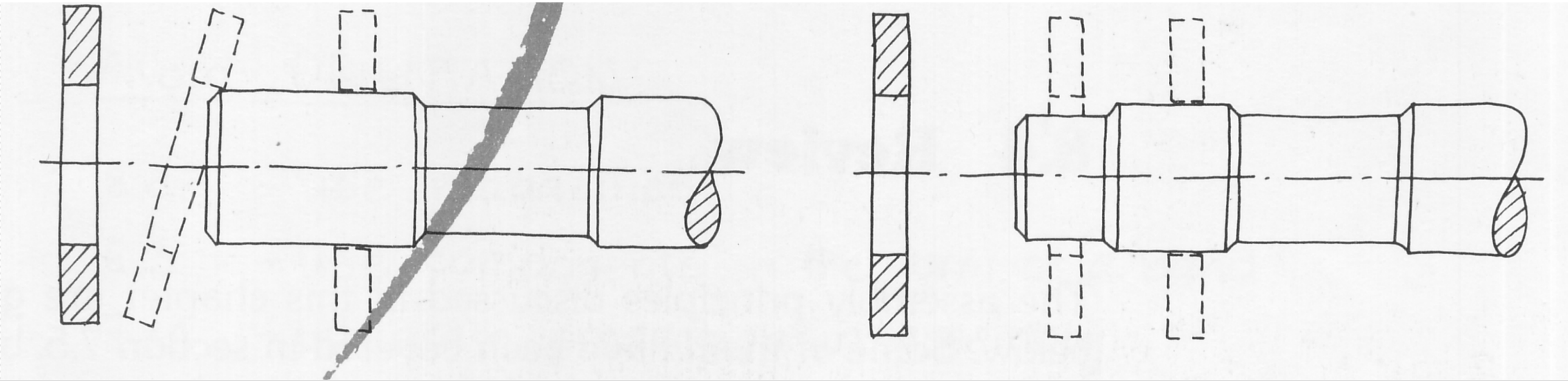
Two subjects where symmetry facilitates orienting.

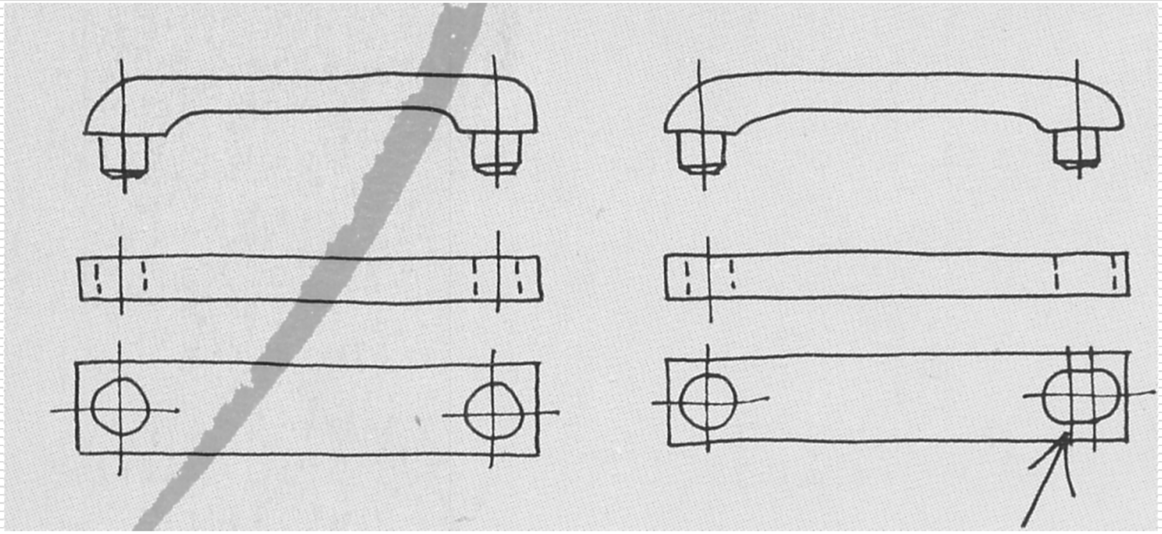
- With hidden features that require a particular orientation, provide an external feature or guide surface to correctly orient the part.





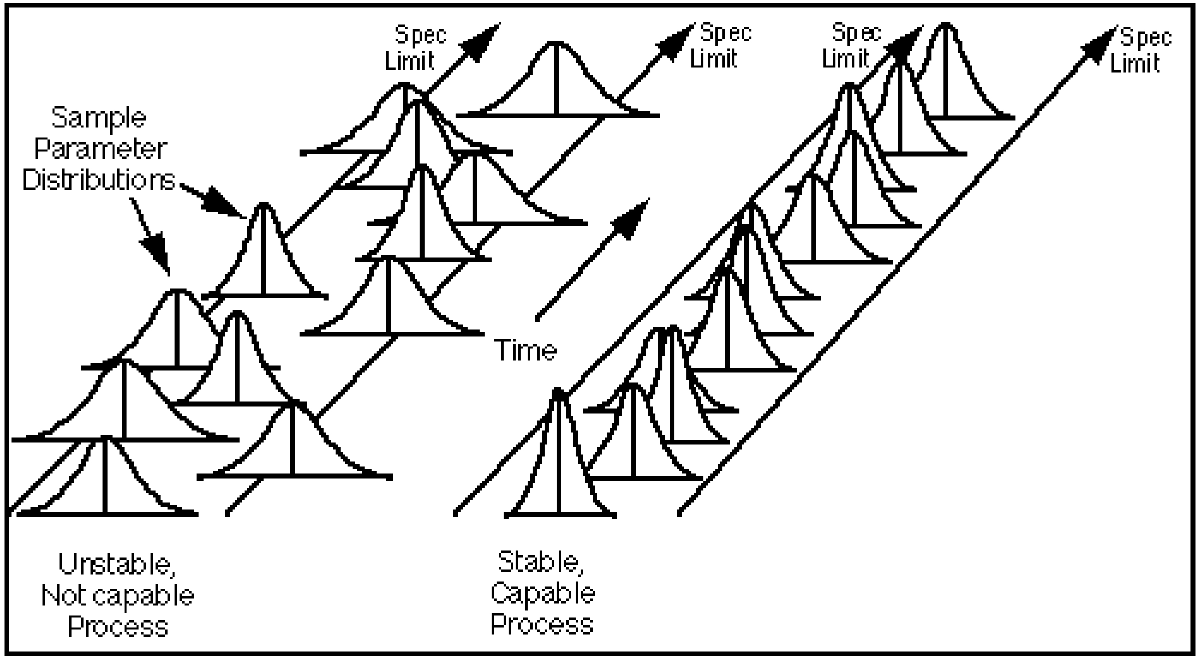
- Guide surfaces should be provided to facilitate insertion.





5. Design Within Process Capabilities

- ❖Process capability is Repeatability and Consistency of a manufacturing process.
- ❖Every equipment has limit .
- ❖ Avoid tight tolerances on multiple, connected parts???



- ❖ Consider mean of range to improve reliability and limit the range of variance.

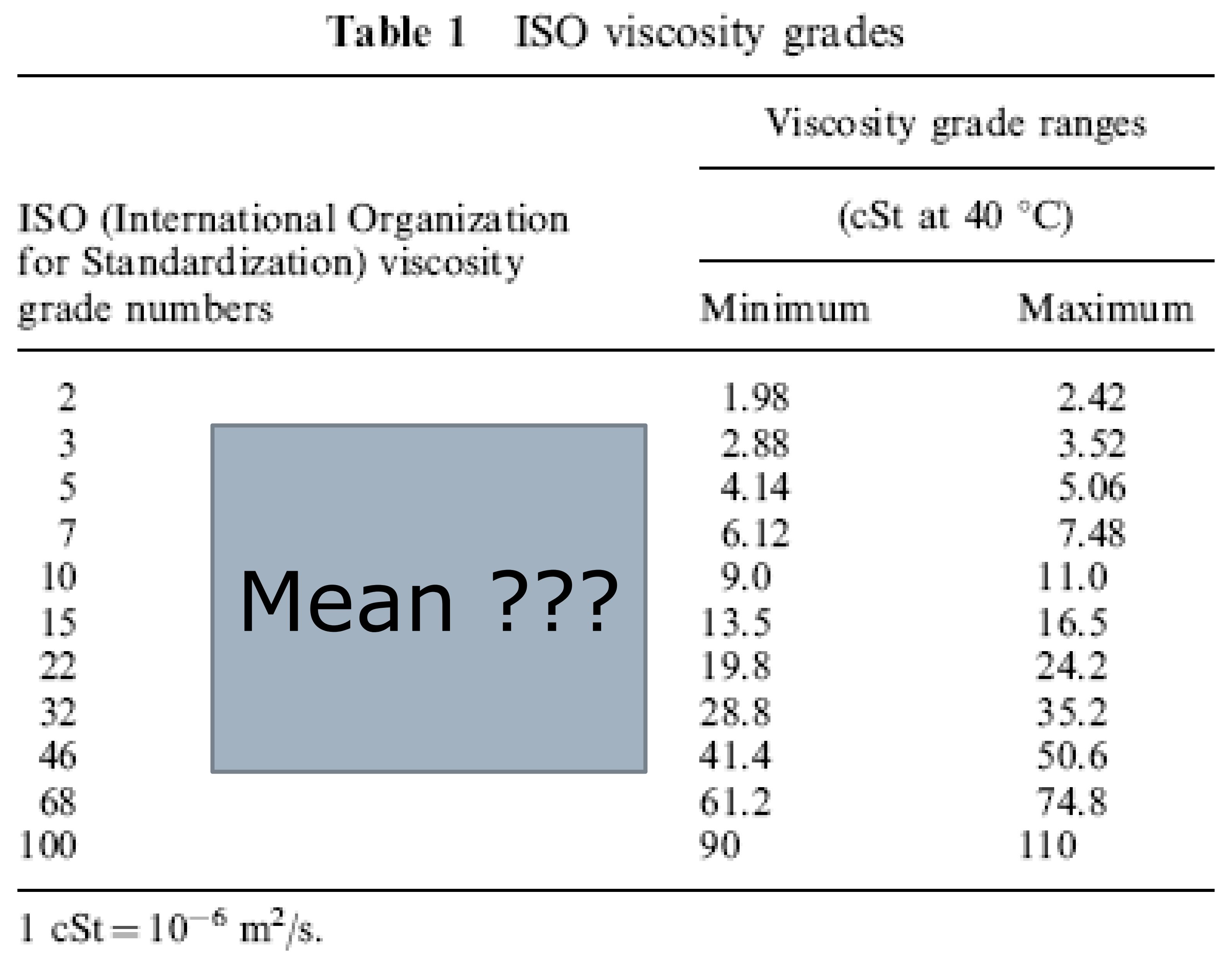


Table 1 ISO viscosity grades

ISO (International Organization for Standardization) viscosity grade numbers	Viscosity grade ranges	
	(cSt at 40 °C)	
	Minimum	Maximum
2	1.98	2.42
3	2.88	3.52
5	4.14	5.06
7	6.12	7.48
10	9.0	11.0
15	13.5	16.5
22	19.8	24.2
32	28.8	35.2
46	41.4	50.6
68	61.2	74.8
100	90	110

1 cSt $= 10^{-6}$ m^2/s.

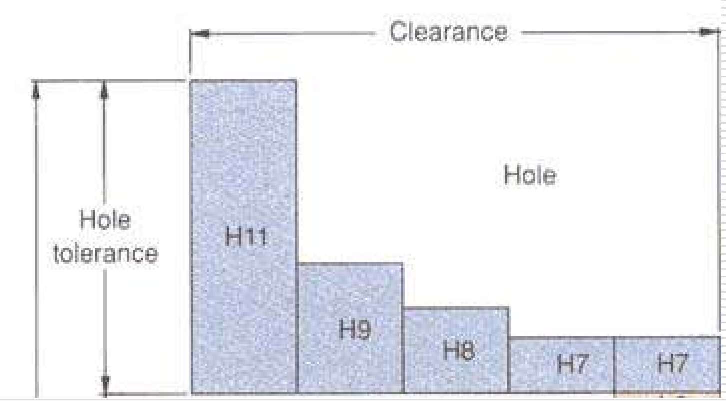
Mean ???

IT Grade	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Lapping	🟦	🟦	🟦	🟦											
Honing		🟦	🟦	🟦											
Super finishing			🟦	🟦	🟦										
Cylindrical grinding			🟦	🟦	🟦	🟦									
Diamond turning			🟦	🟦	🟦	🟦									
Plan grinding				🟦	🟦	🟦	🟦	🟦							
Broaching				🟦	🟦	🟦	🟦	🟦							
Reaming				🟦	🟦	🟦	🟦	🟦							
Boring, Turning					🟦	🟦	🟦	🟦	🟦	🟦	🟦				
Sawing								🟦	🟦	🟦					
Milling								🟦	🟦	🟦	🟦	🟦			
Planning, Shaping									🟦	🟦	🟦	🟦	🟦		
Extruding									🟦	🟦	🟦	🟦			
Cold Rolling, Drawing									🟦	🟦	🟦	🟦	🟦		
Drilling										🟦	🟦	🟦	🟦		
Die Casting											🟦	🟦	🟦	🟦	
Forging												🟦	🟦	🟦	🟦
Sand Casting													🟦	🟦	🟦
Hot rolling, Flame cutting													🟦	🟦	🟦

Mean ???

	Nominal Sizes (mm)										
over	1	3	6	10	18	30	50	80	120	180	250
inc.	3	6	10	18	30	50	80	120	180	250	315
IT Grade											
1	0.8	1	1	1.2	1.5	1.5	2	2.5	3.5	4.5	6
2	1.2	1.5	1.5	2	2.5	2.5	3	4	5	7	8
3	2	2.5	2.5	3	4	4	5	6	8	10	12
4	3	4	4	5	6	7	8	10	12	14	16
5	4	5	6	8	9	11	13	15	18	20	23
6	6	8	9	11	13	16	19	22	25	29	32
7	10	12	15	18	21	25	30	35	40	46	52
8	14	18	22	27	33	39	46	54	63	72	81
9	25	30	36	43	52	62	74	87	100	115	130
10	40	48	58	70	84	100	120	140	160	185	210
11	60	75	90	110	130	160	190	220	250	290	320
12	100	120	150	180	210	250	300	350	400	460	520
13	140	180	220	270	330	390	460	540	630	720	810
14	250	300	360	430	520	620	740	870	1000	1150	1300

Examples



Hole 110H11

Minimum = 110mm + 0mm = 110.000mm ...

Maximum = 110mm + (0+0.220) = 110.220mm

Resulting limits 110.000/110.220

Tolerance of hub, t_{lh}=220μm

Shaft 110e9...

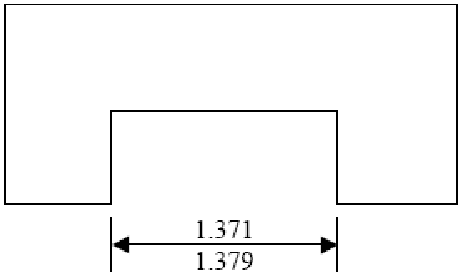
Maximum = 110mm – 0.072=109.928mm...

Minimum = 110mm - (0.072 +0.087) = 109.841mm

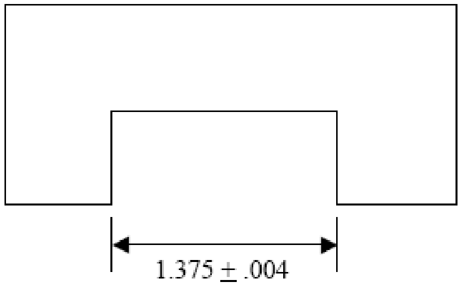
Resulting limits 109.841/ 109.928

Tolerance of shaft, t_{ls}=87μm

Tolerances



Limit Dimensioning

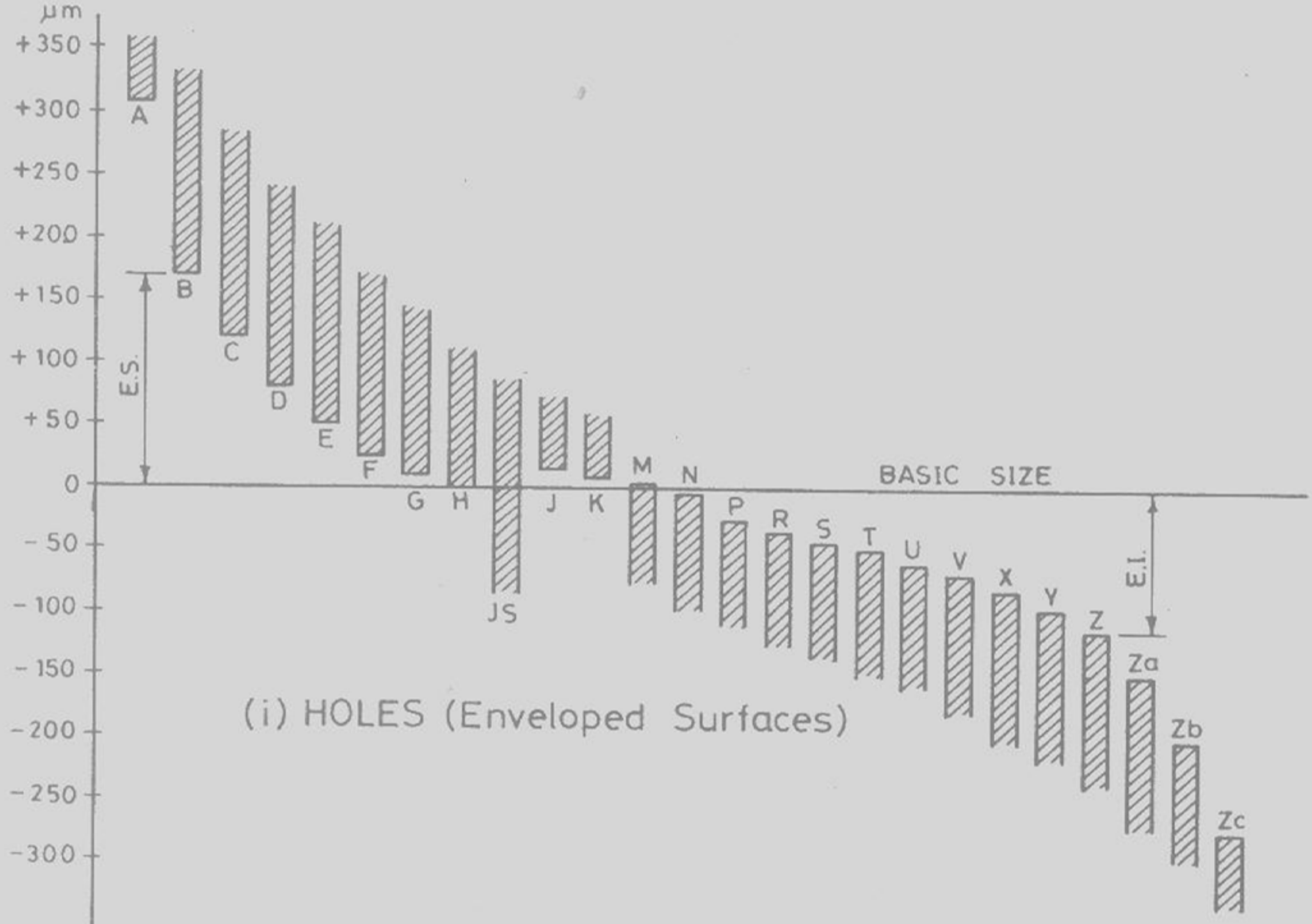


Plus & Minus Tolerancing

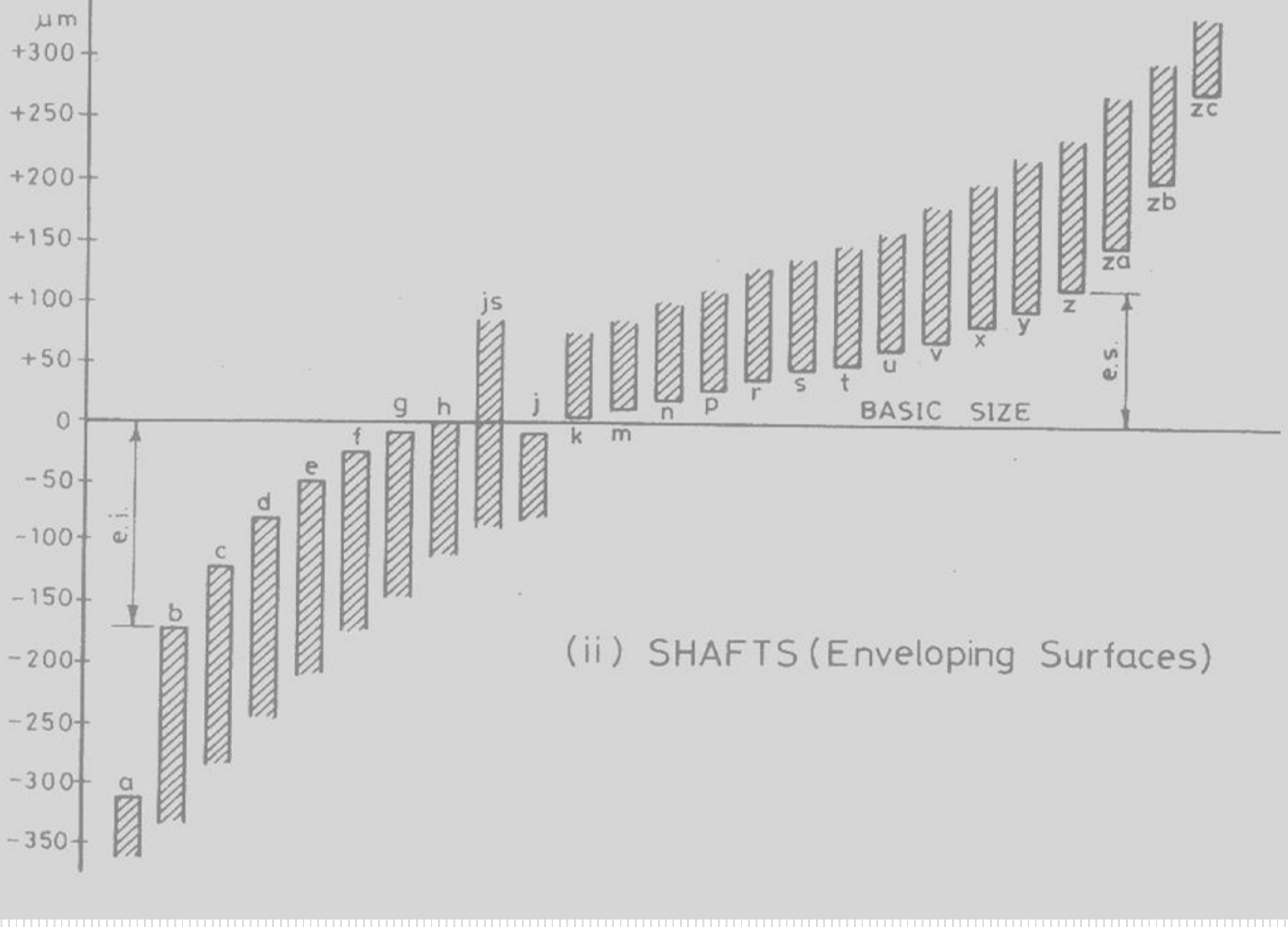
- Machine elements are manufactured / fabricated with some tolerance on their basic (normal size, i.e. ϕ 20mm) dimensions--economize the manufacturing process
  - Tolerance: “permissible variation in the dimensions of a component”.
  - Tolerance: Unilateral or bilateral.

$$20^{+0.04}_{0.00} \qquad 20^{0.00}_{-0.04} \qquad 20^{+0.04}_{-0.02} \qquad 20^{\pm 0.03}$$

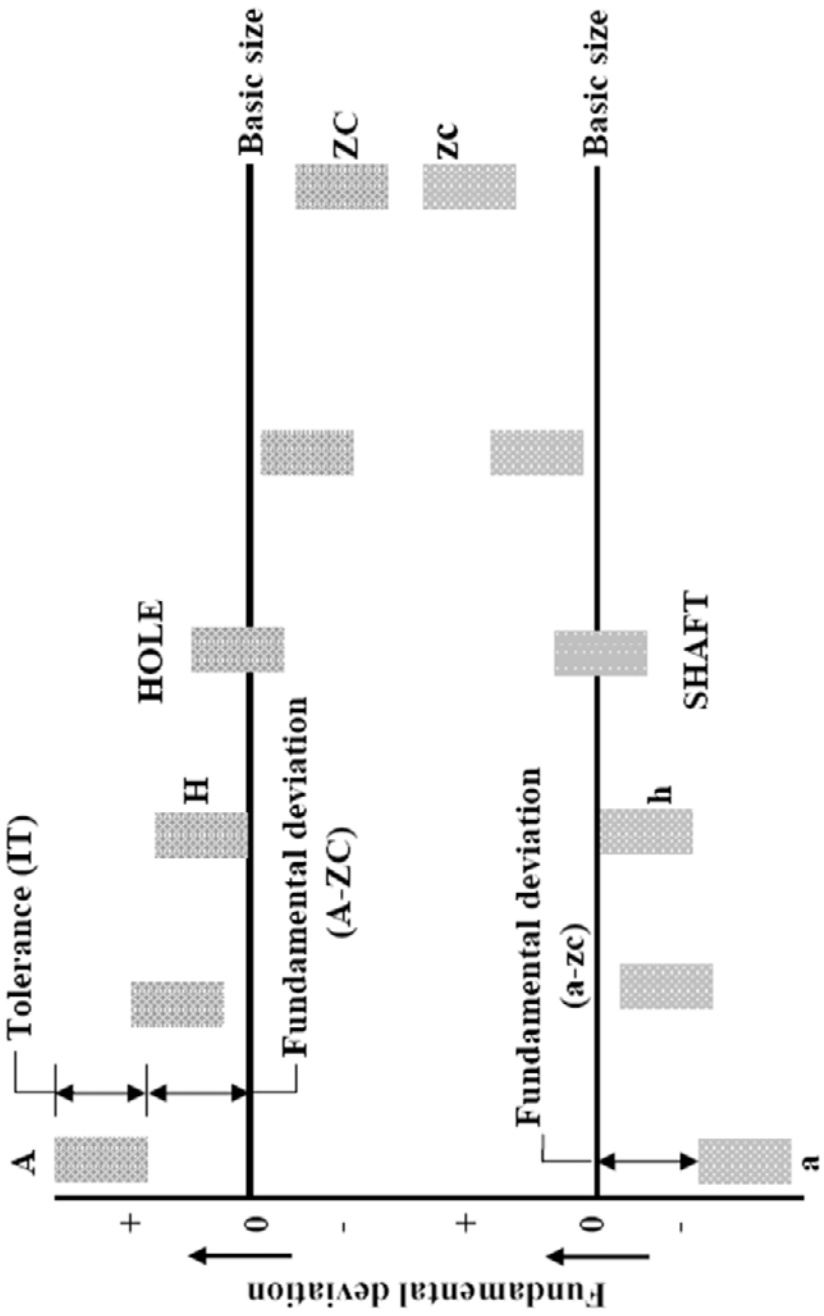
$$\mu_d = 20; \sigma_d = 0.01$$



(i) HOLES (Enveloped Surfaces)



(ii) SHAFTS (Enveloping Surfaces)

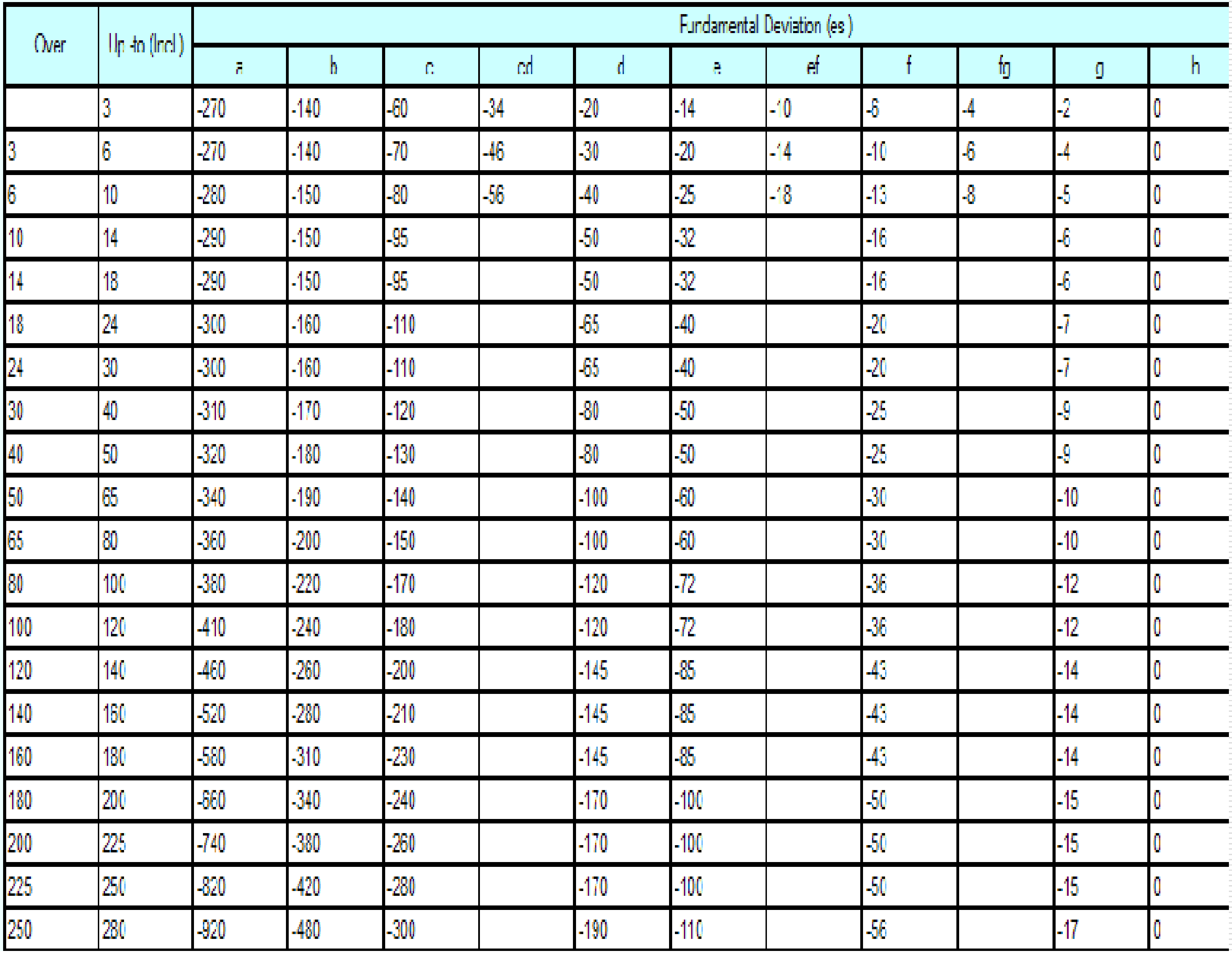


Tolerance is denoted as IT and it has 18 grades; greater the number, more is the tolerance limit.

The fundamental deviations for the hole are denoted by capital letters from A to ZC, having altogether 25 divisions.

Similarly, the fundamental deviations for the shaft is denoted by small letters from a to zc.

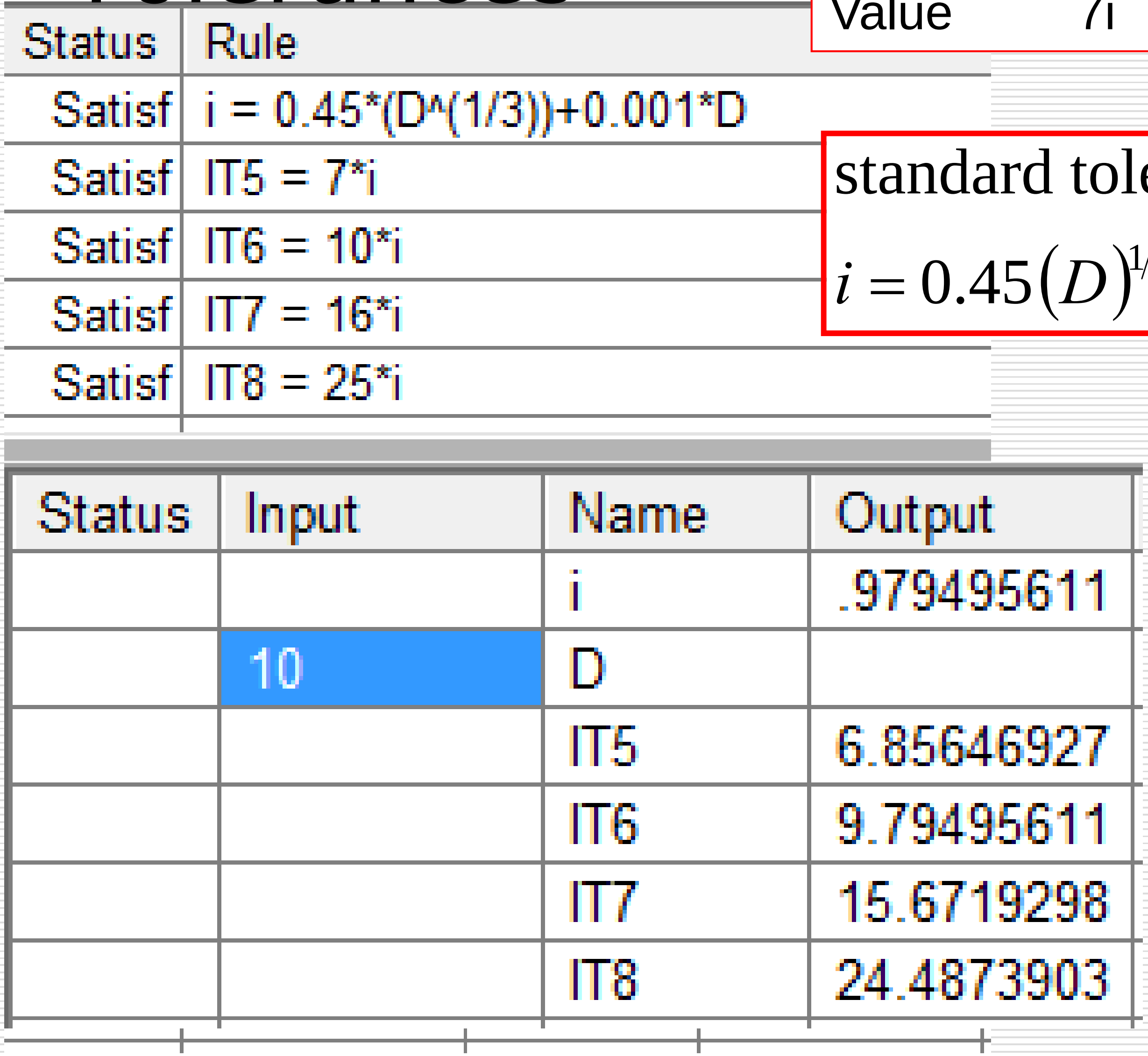
over	Up to (Incl.)	Fundamental Deviation (EI)										
		A	B	C	CD	D	E	EF	F	FG	G	H
	3	270	140	60	34	20	14	10	6	4	2	0
3	6	270	140	70	46	30	20	14	10	6	4	0
6	10	280	150	80	56	40	25	18	13	8	5	0
10	14	290	150	95		50	32		16		6	0
14	18	290	150	95		50	32		16		6	0
18	24	300	160	110		65	40		20		7	0
24	30	300	160	110		65	40		20		7	0
30	40	310	170	120		80	50		25		9	0
40	50	320	180	130		80	50		25		9	0
50	65	340	190	140		100	60		30		10	0
65	80	360	200	150		100	60		30		10	0
80	100	380	220	170		120	72		36		12	0
100	120	410	240	180		120	72		36		12	0
120	140	460	260	200		145	85		43		14	0
140	160	520	280	210		145	85		43		14	0
160	180	580	310	230		145	85		43		14	0
180	200	660	340	240		170	100		50		15	0
200	225	740	380	260		170	100		50		15	0
225	250	820	420	280		170	100		50		15	0
250	280	920	480	300		190	110		56		17	0



Over	Up to (Incl)	Fundamental Deviation (es)										
		a	b	c	cd	d	e	ef	f	fg	g	h
	3	-270	-140	-60	-34	-20	-14	-10	-6	-4	-2	0
3	6	-270	-140	-70	-46	-30	-20	-14	-10	-6	-4	0
6	10	-280	-150	-80	-56	-40	-25	-18	-13	-8	-5	0
10	14	-290	-150	-95		-50	-32		-16		-6	0
14	18	-290	-150	-95		-50	-32		-16		-6	0
18	24	-300	-160	-110		-65	-40		-20		-7	0
24	30	-300	-160	-110		-65	-40		-20		-7	0
30	40	-310	-170	-120		-80	-50		-25		-9	0
40	50	-320	-180	-130		-80	-50		-25		-9	0
50	65	-340	-190	-140		-100	-60		-30		-10	0
65	80	-360	-200	-150		-100	-60		-30		-10	0
80	100	-380	-220	-170		-120	-72		-36		-12	0
100	120	-410	-240	-180		-120	-72		-36		-12	0
120	140	-460	-260	-200		-145	-85		-43		-14	0
140	160	-520	-280	-210		-145	-85		-43		-14	0
160	180	-580	-310	-230		-145	-85		-43		-14	0
180	200	-660	-340	-240		-170	-100		-50		-15	0
200	225	-740	-380	-260		-170	-100		-50		-15	0
225	250	-820	-420	-280		-170	-100		-50		-15	0
250	280	-920	-480	-300		-190	-110		-56		-17	0

Tolerances

IT grade	5	6	7	8
Value	7i	10i	16i	25i



Status	Rule
Satisf	i = 0.45*(D^(1/3))+0.001*D
Satisf	IT5 = 7*i
Satisf	IT6 = 10*i
Satisf	IT7 = 16*i
Satisf	IT8 = 25*i

Status	Input	Name	Output
		i	.979495611
	10	D	
		IT5	6.85646927
		IT6	9.79495611
		IT7	15.6719298
		IT8	24.4873903

standard tolerance unit, i

$$i = 0.45(D)^{1/3} + 0.001D$$

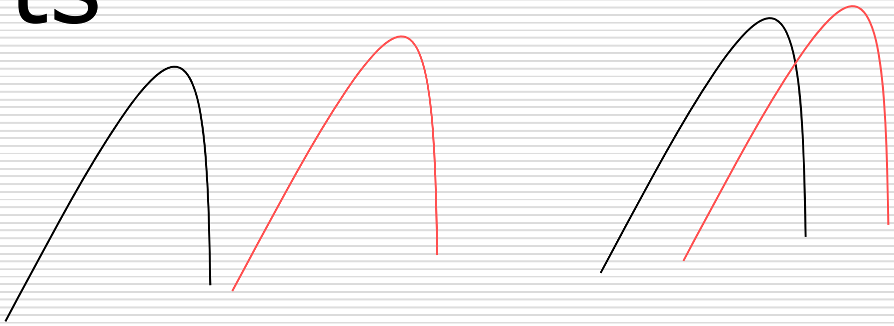
Examples 34H11/c11

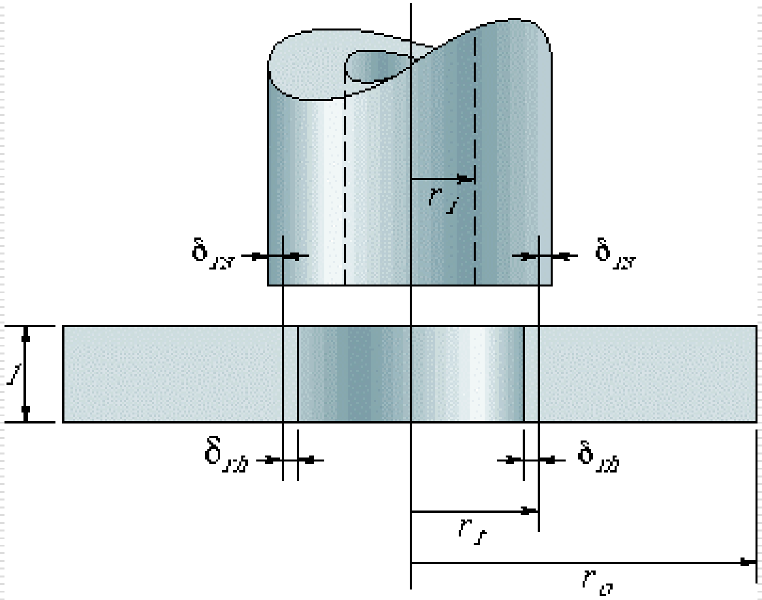
Hole 34H11
Minimum = 34mm + 0mm = 34.000mm ...
Maximum = 34mm + (0+0.160) =   34.160mm
Resulting limits 34.000/34.160
Tolerance of hub, t_{lh}=160μm

Shaft 34c11...
Maximum = 34mm – 0.120=33.880mm...
Minimum = 34mm - (0.120 +0.160) = 33.720mm
Resulting limits 33.880/ 33.720
Tolerance of shaft, t_{ls}=160 μm

Wide range of tolerance → transition fit

Fits



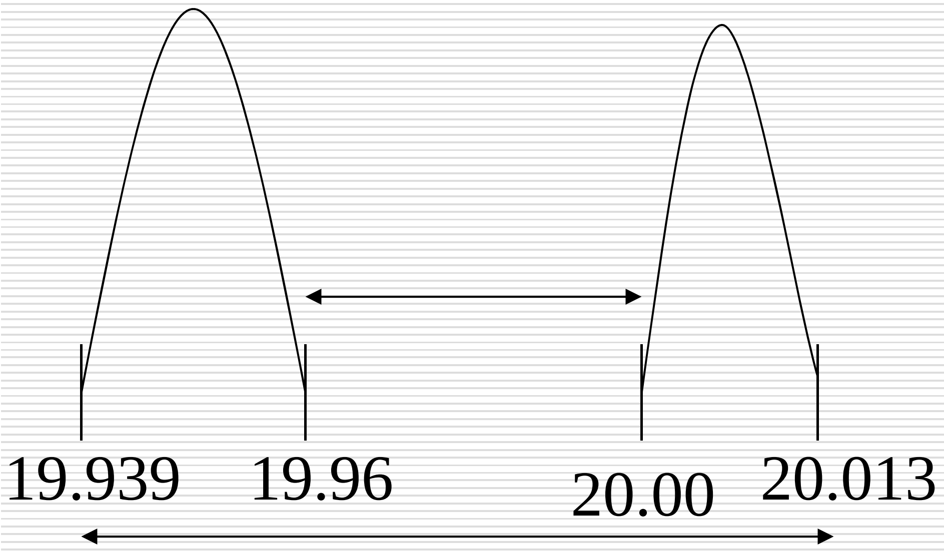


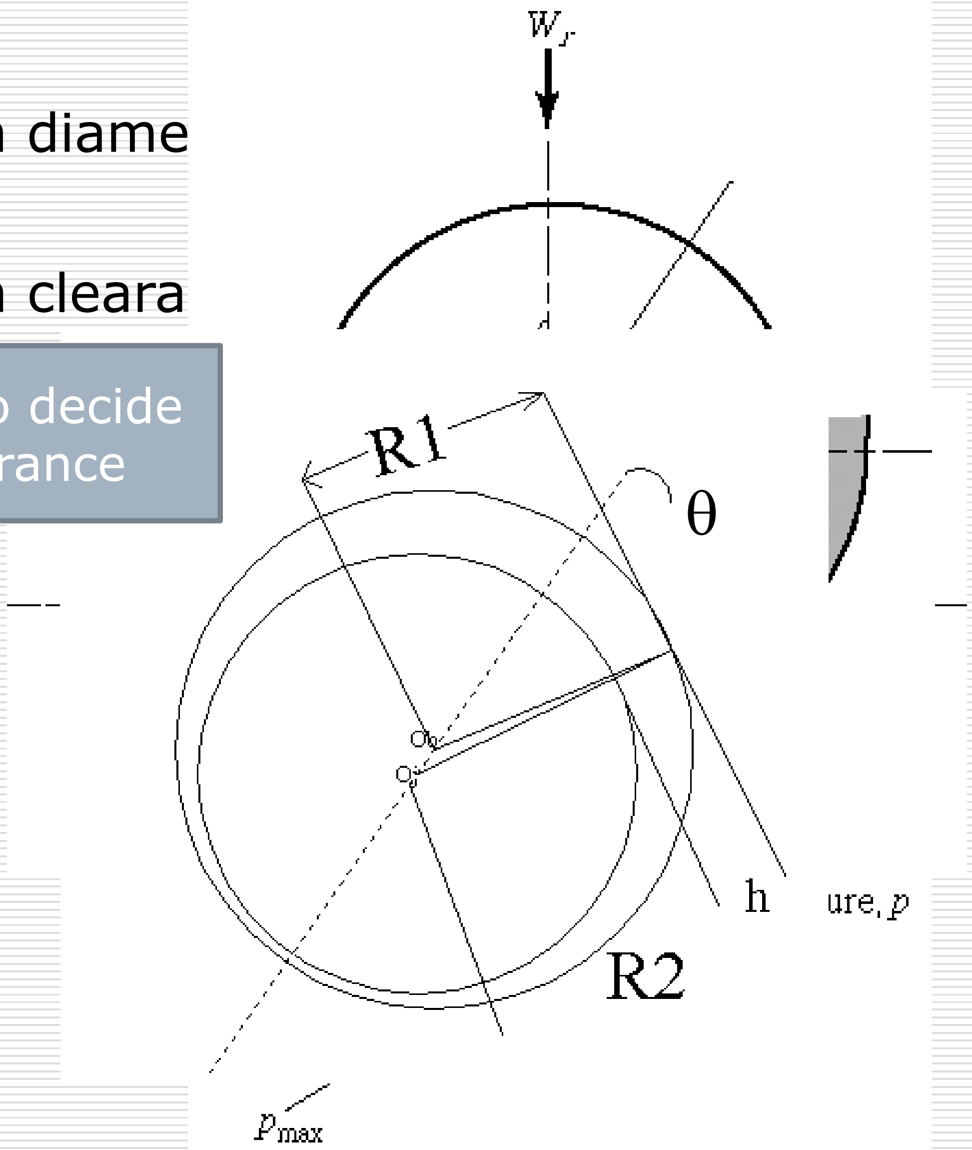
- Careful decision on tolerance is important for assembling two components.
  - Relationship resulting from difference in sizes of components before assembly is called a “Fit”.
  - <u>Clearance fit</u>: positive gap between hole and shaft. Relative movement is possible.
  - <u>Interference fit</u>: Negative gap. Relative movement is restricted.
  - <u>Transition fit</u>: border case. Either a clearance or interference fit, depending upon actual values of dimensions of mating components.

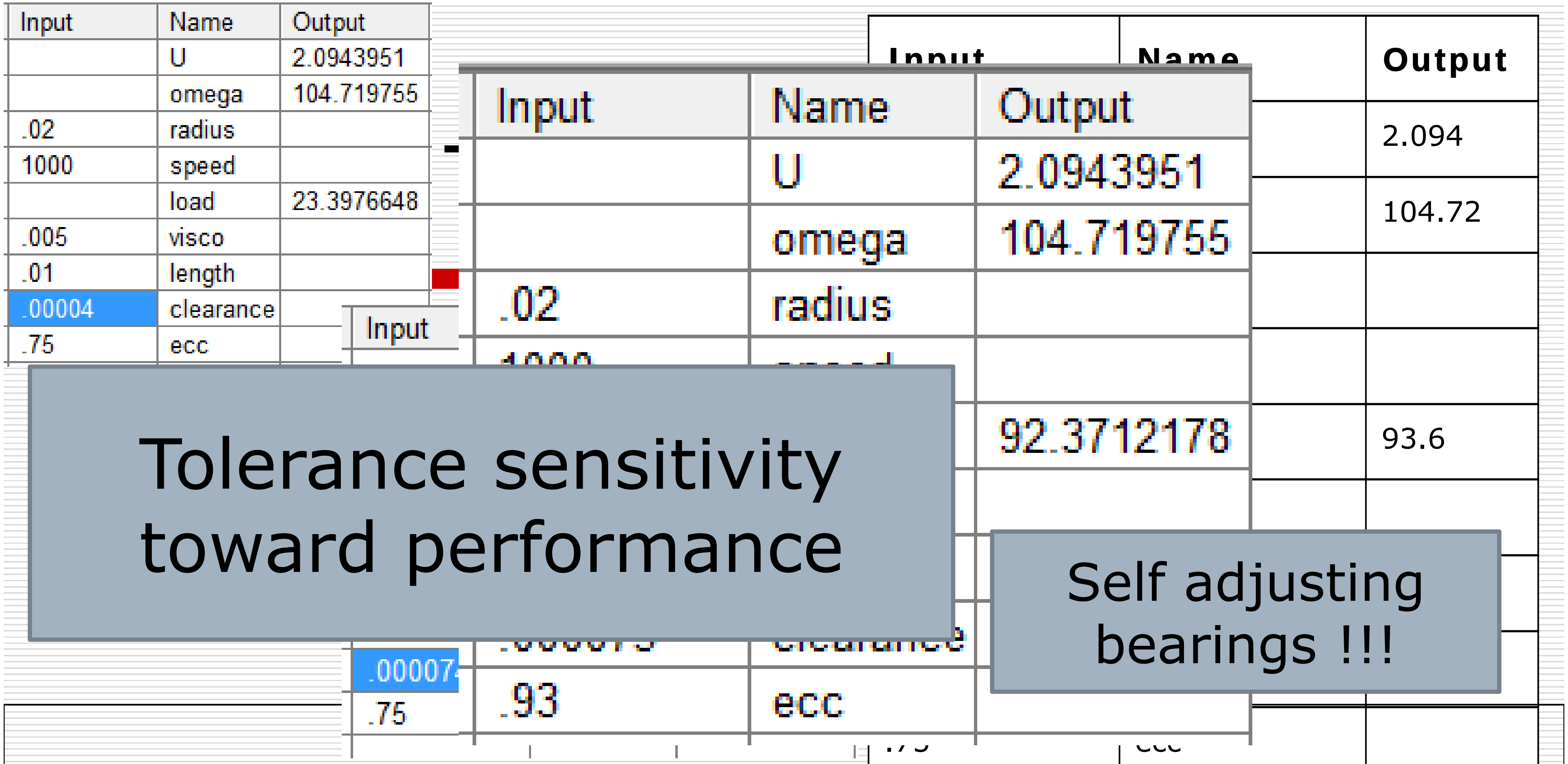
Prob : A bearing $\left(20^{+0.013}_{\ 0.000}\right)$ and a crank-pin $\left(20^{-0.040}_{-0.061}\right)$ are assembled. Calculate :

- Maximum and minimum diame
bearing.
- Maximum and minimum cleara
and bearing.

How to decide clearance







Input	Name	Output
	U	2.0943951
	omega	104.719755
.02	radius	
1000	speed	
	load	23.3976648
.005	visco	
.01	length	
.00004	clearance	
.75	ecc	

Input	Name	Output
	U	2.0943951
	omega	104.719755
.02	radius	
		92.3712178
.93	ecc	

Input
.00007
.75

Input	Name	Output
		2.094
		104.72
		93.6

Tolerance sensitivity toward performance

Self adjusting bearings !!!

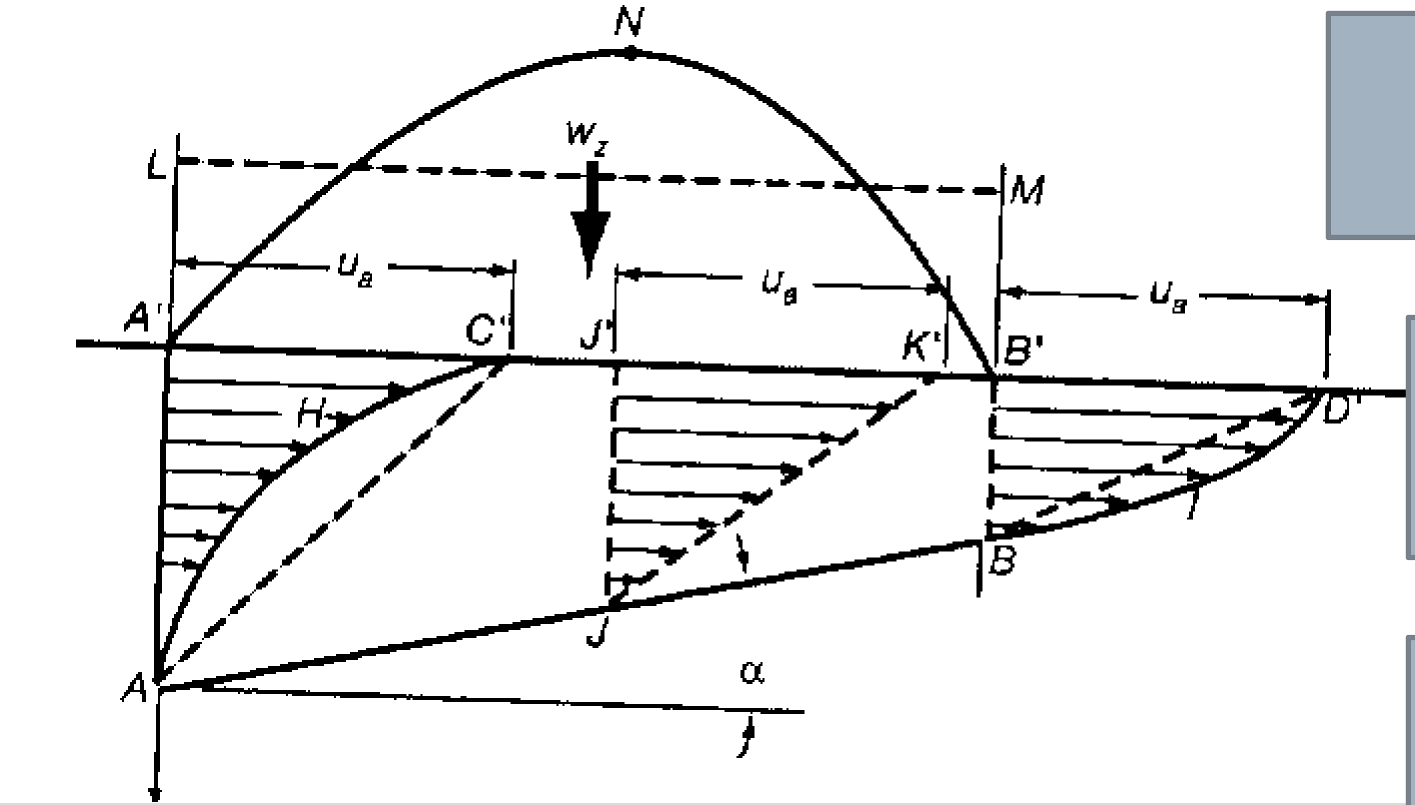
U = omega * radius

omega = 2 *pi()/60 * speed

load = U * visco * (length ^ 3) /(clearance ^2) * pi()/4 * ecc/((1-ecc^2)^2) *sqrt((16/(pi()^2)-1)*(ecc^2) + 1)

clearance = 0.001 * radius

How do I get load expression



$$\frac{\partial}{\partial \mathrm{x}}\left\{\frac{\mathrm{h}^3}{12\eta}\frac{\partial P}{\partial x}\right\}+\frac{\partial}{\partial \mathrm{z}}\left\{\frac{\mathrm{h}^3}{12\eta}\frac{\partial P}{\partial z}\right\}=\frac{1}{2}\frac{\partial}{\partial \mathrm{x}}\{(U_2-U_1)h\}+\left(V_h-V_0\right)+\frac{1}{2}\frac{\partial}{\partial \mathrm{z}}\{(W_2-W_1)h\} \quad (14)$$

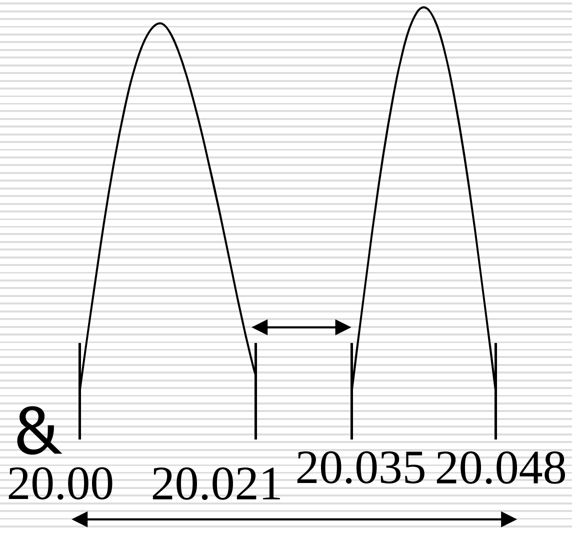
http://web.iitd.ac.in/~hirani/mel417.pdf

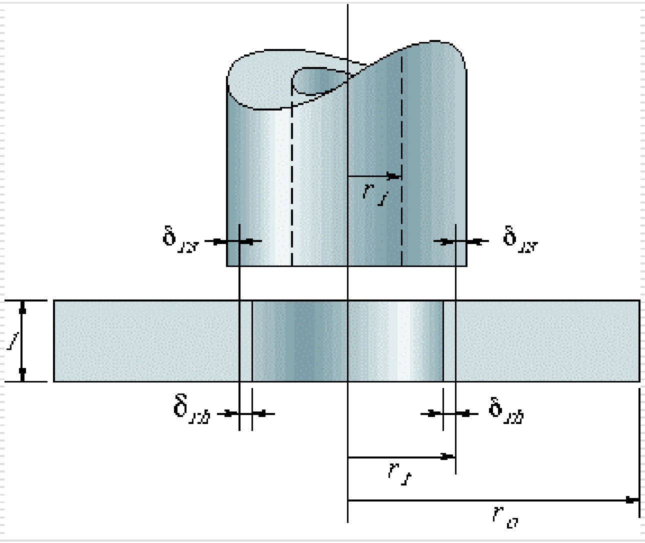
Prob : A shaft $(20^{+0.048}_{+0.035})$ is inserted in a housing $(20^{+0.021}_{0.000})$.

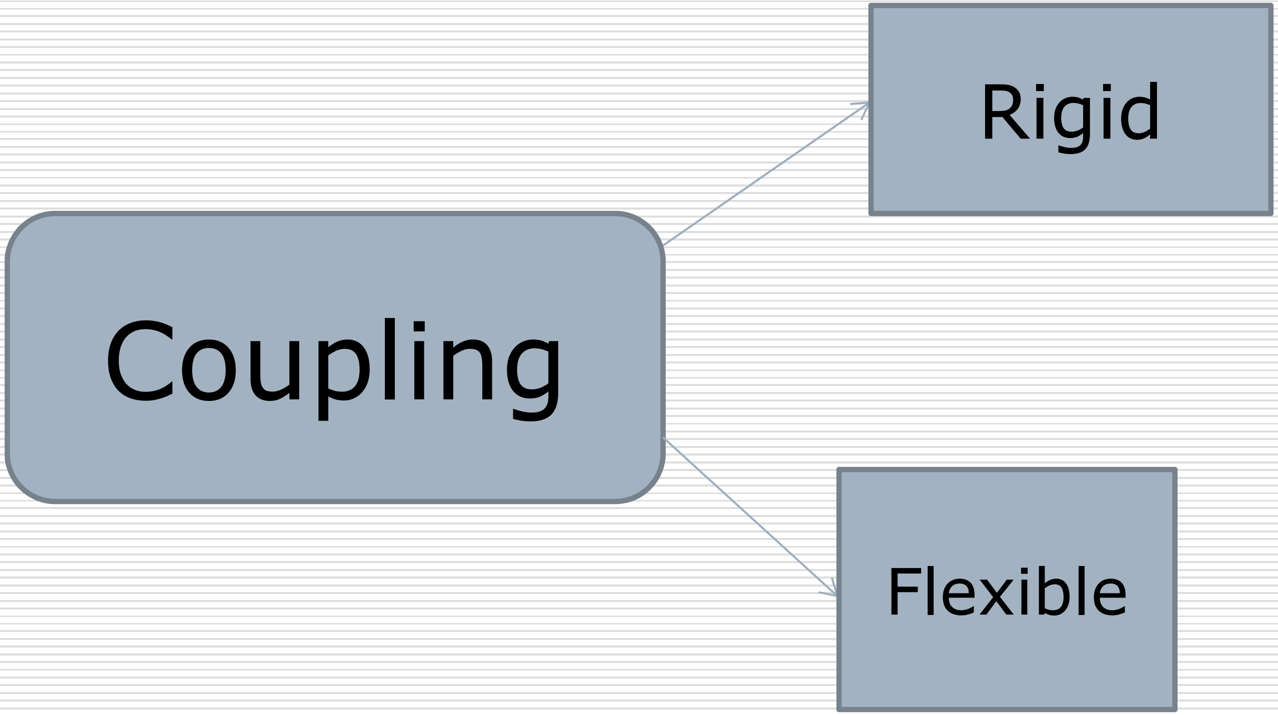
Calculate:

- Maximum and minimum diameters of the shaft and housing-hole.
- Maximum and minimum interference between the shaft and its housing.

Find stresses caused by interference between shaft & hub.

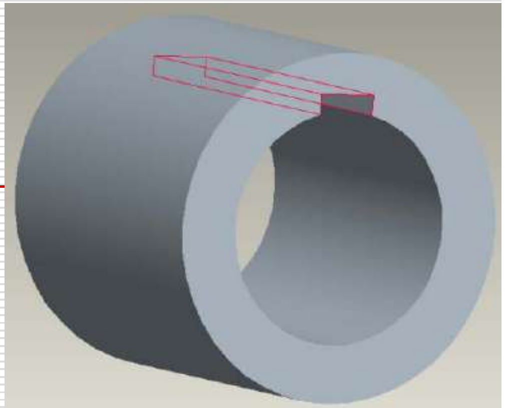






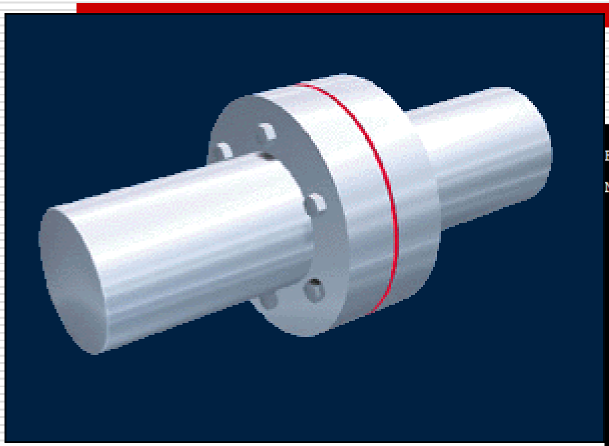
Coupling

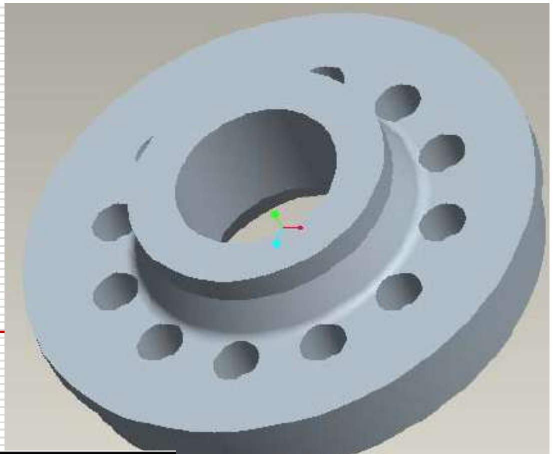
Rigid

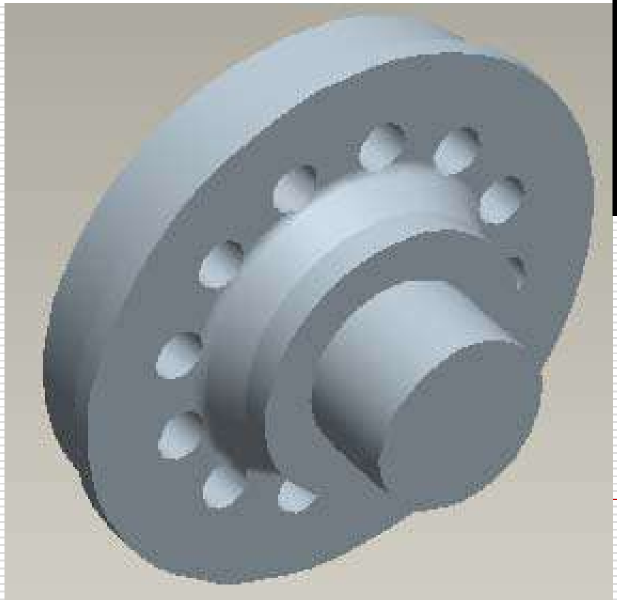


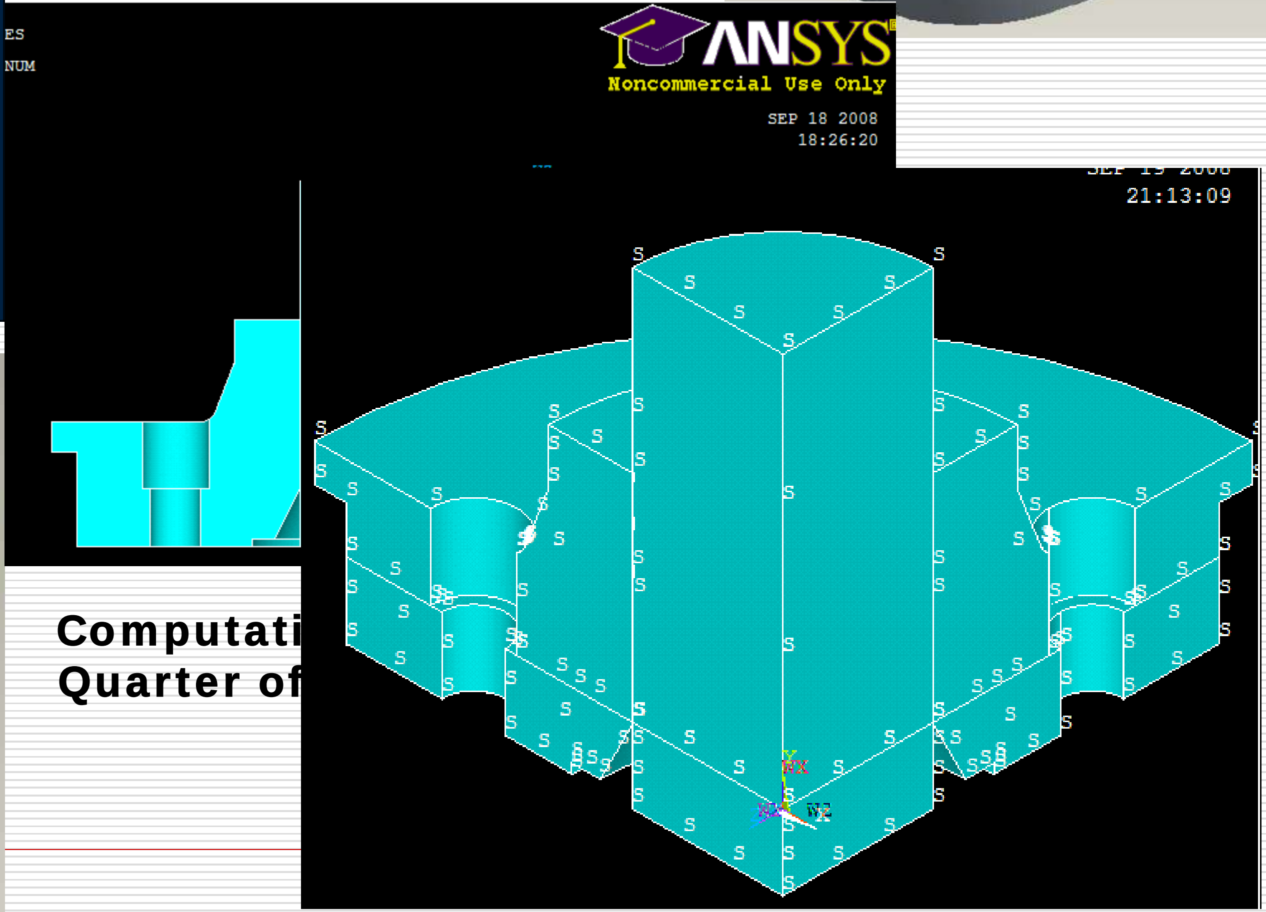
Flexible

Analysis of Steam Turbine Coupling

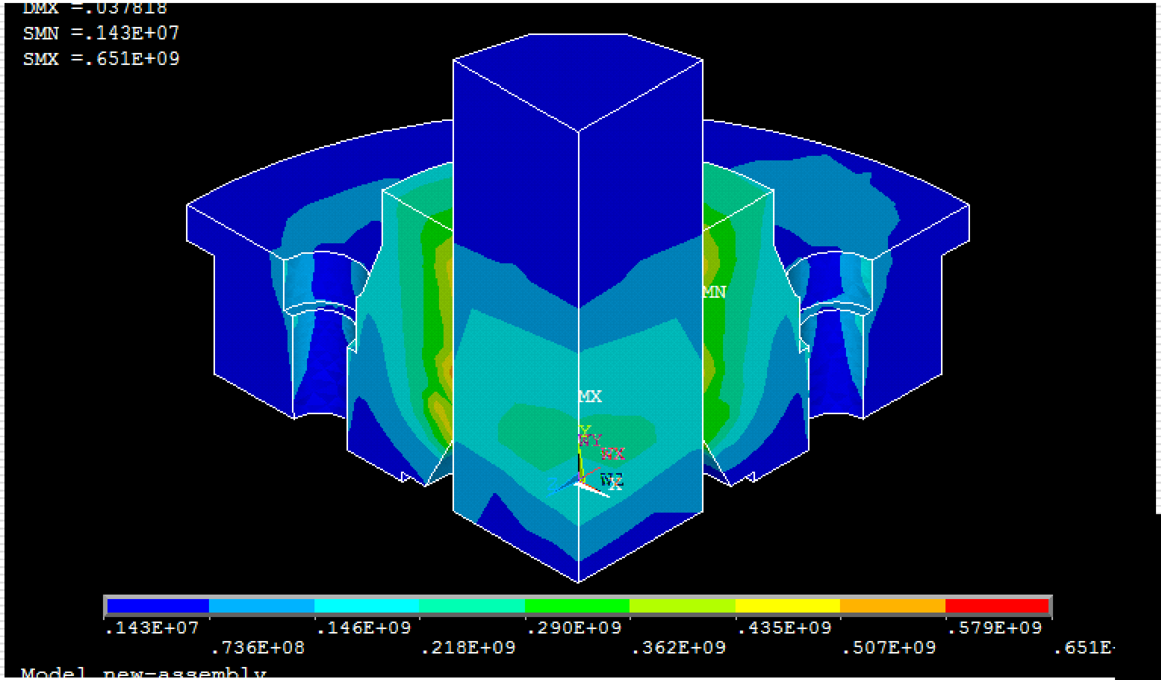




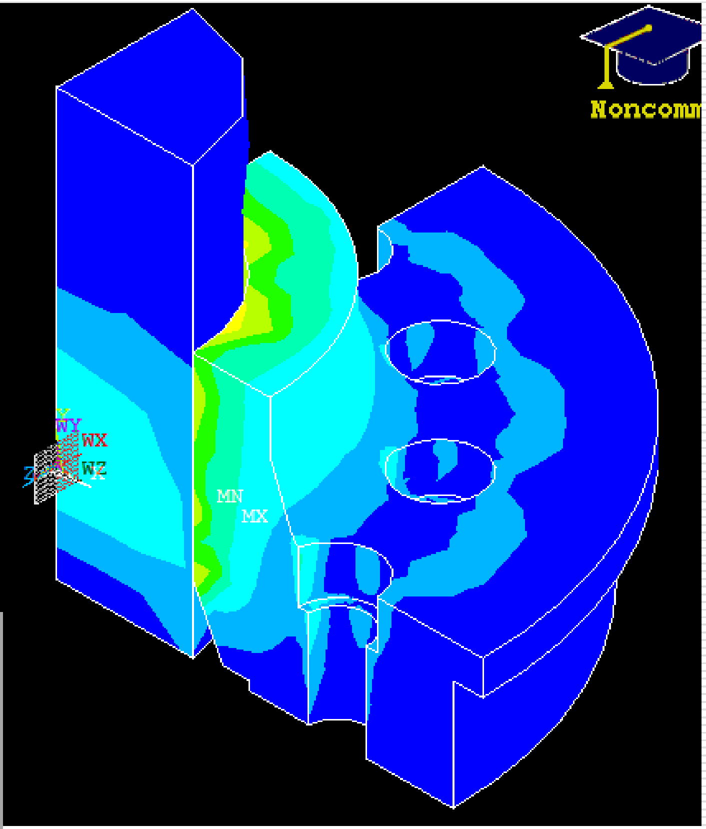




Computati
Quarter of



Von-Mises stress distribution



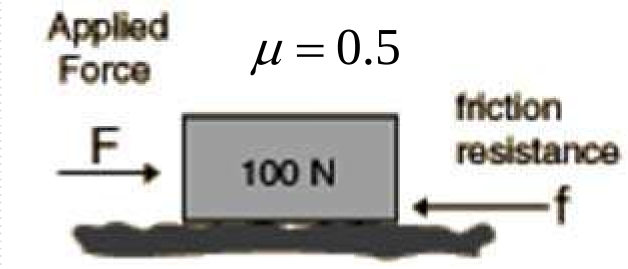
MATERIAL PROPERTIES

Young’s modulus = 2.1×10^{11} Pa.
Poisson’s ratio = 0.29
Density = 7850 kg/m^3
Friction coefficient = 0.12

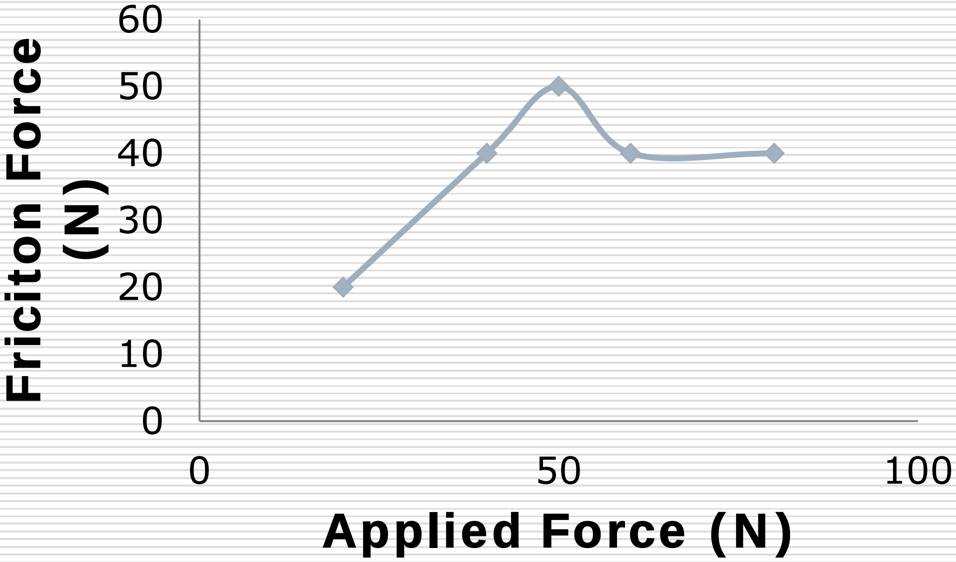
Design of COUPLING between Turbines .

Rotor dia =310 mm.
Max power = 330 MW
Speed = 3000 rpm

Friction… Statistical !!!!

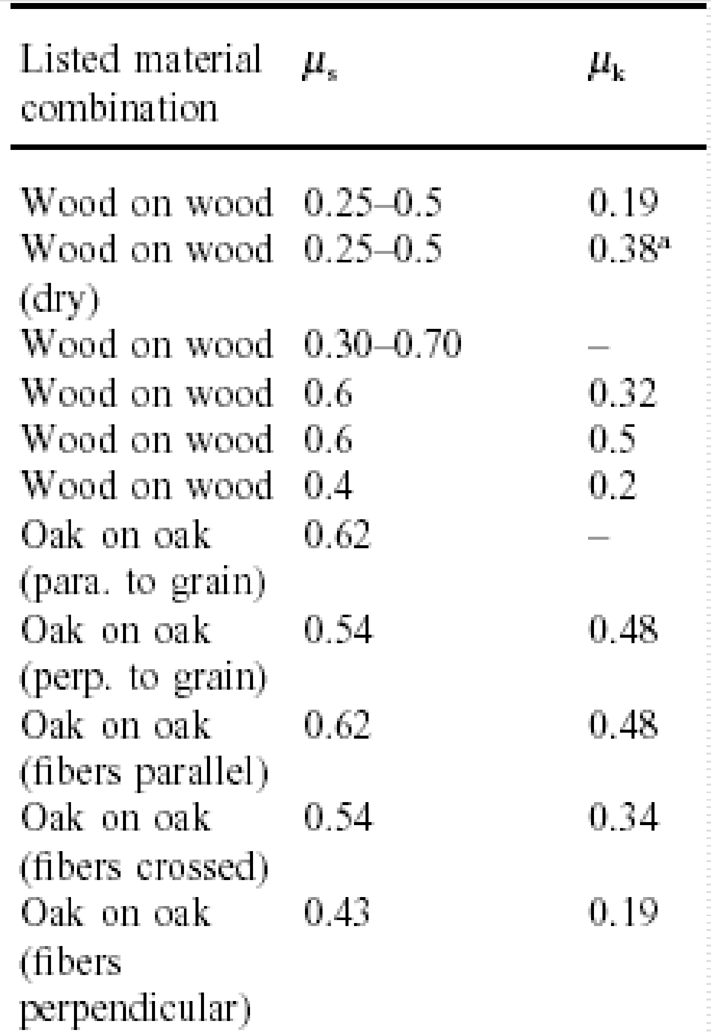


$\mu = 0.5$



Difference between the static and kinetic friction may initiate ‘stick-slip’.

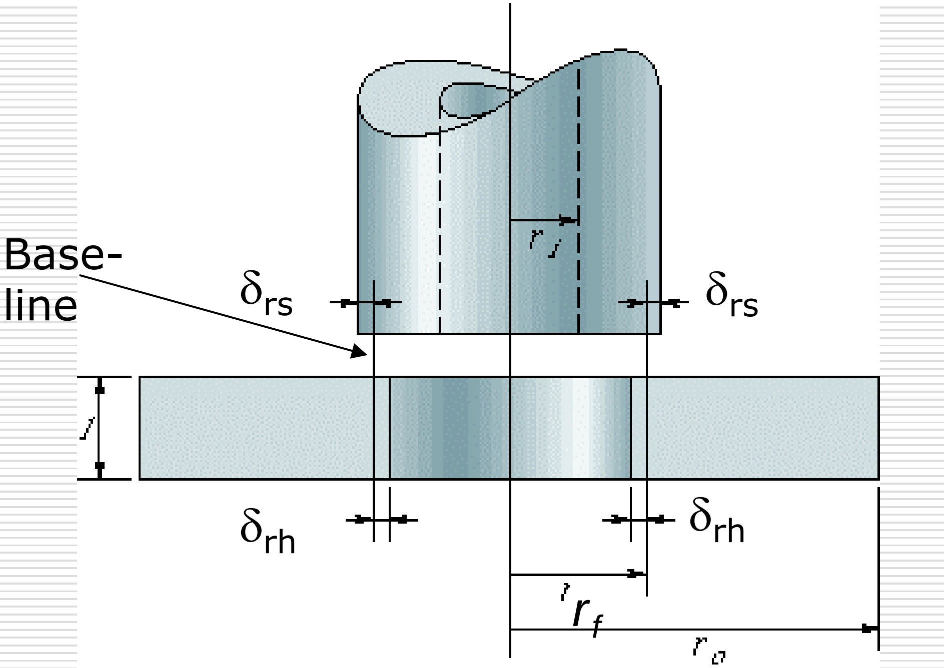
μ for wood-on-wood reported in various articles.

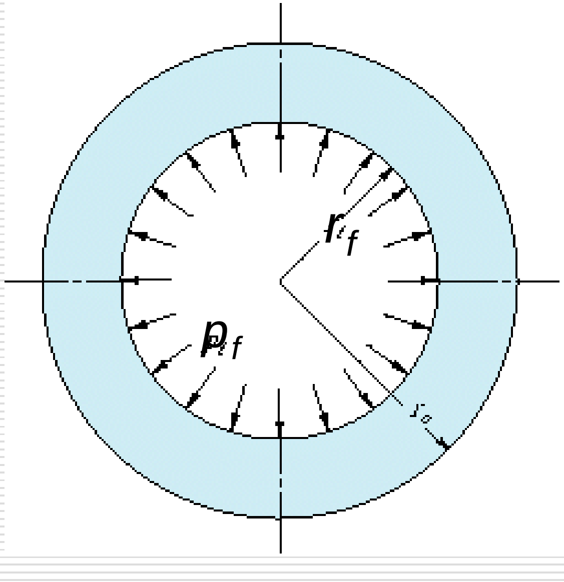


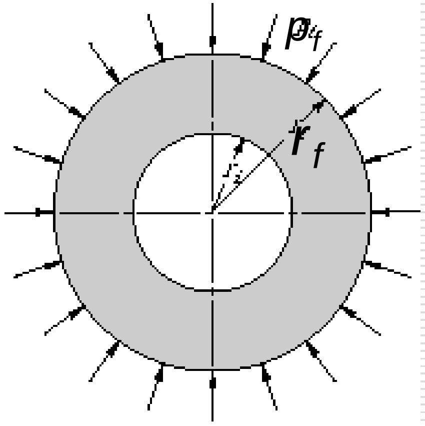
Listed material combination	μ_s	μ_k
Wood on wood	0.25–0.5	0.19
Wood on wood (dry)	0.25–0.5	0.38[a]
Wood on wood	0.30–0.70	–
Wood on wood	0.6	0.32
Wood on wood	0.6	0.5
Wood on wood	0.4	0.2
Oak on oak (para. to grain)	0.62	–
Oak on oak (perp. to grain)	0.54	0.48
Oak on oak (fibers parallel)	0.62	0.48
Oak on oak (fibers crossed)	0.54	0.34
Oak on oak (fibers perpendicular)	0.43	0.19

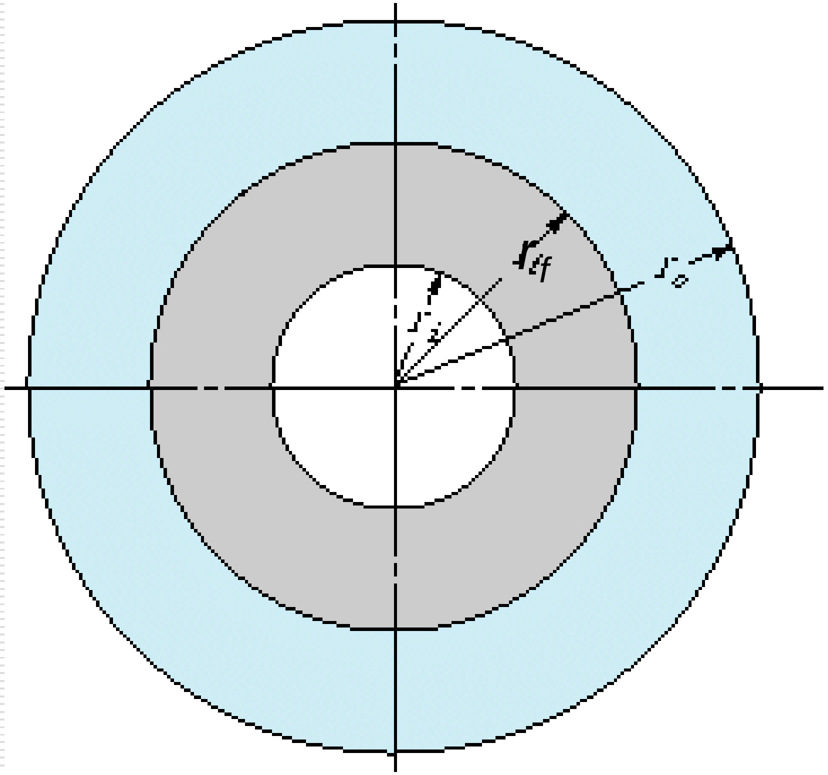
Press Fit

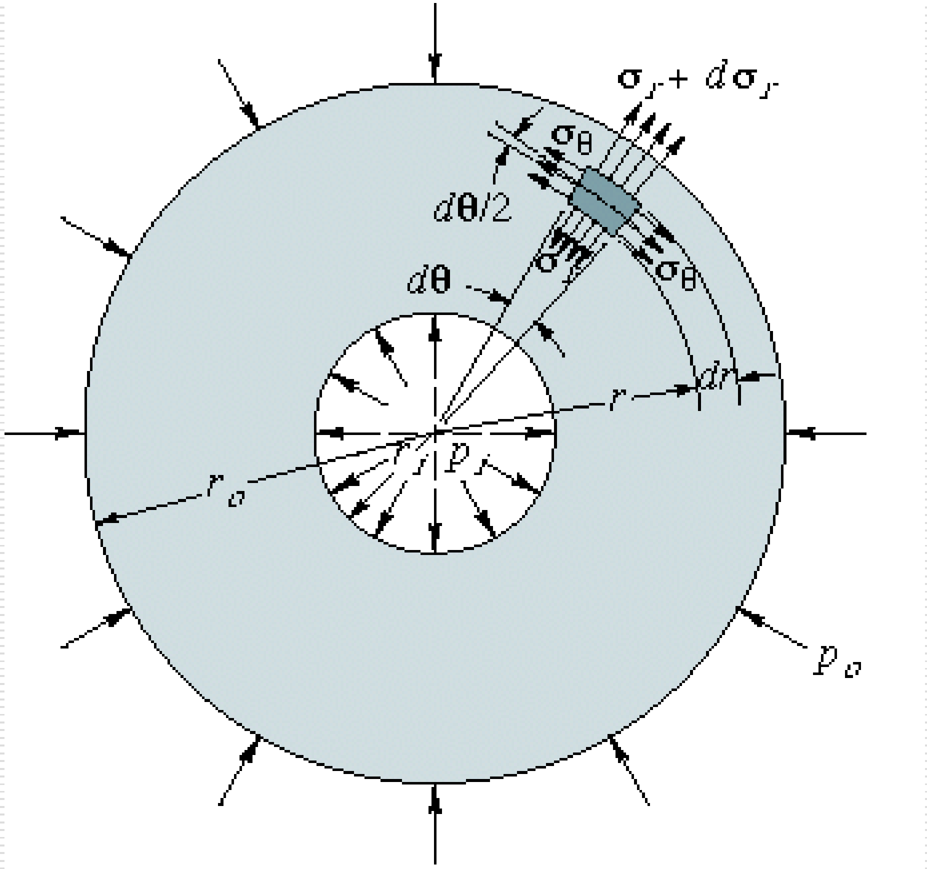
Pressure p_f is caused by interference between shaft & hub. Pressure increases radius of hole and decreases radius of shaft.

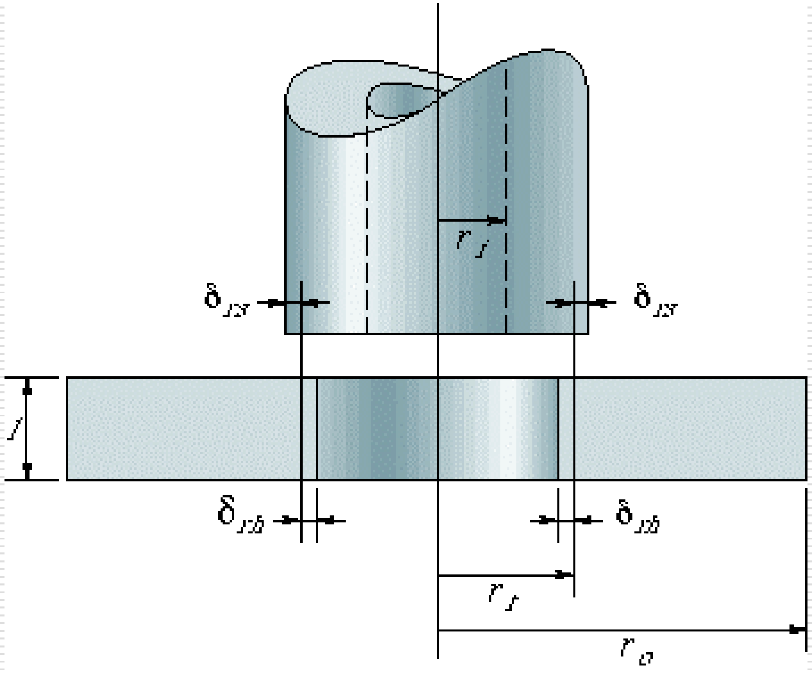












$$\text{Circumferential strain } \varepsilon_\theta = \frac{(r+\delta_r)\,d\theta - r\,d\theta}{r\,d\theta} = \frac{\delta_r}{r} = \frac{(\sigma_\theta - \nu\,\sigma_r)}{E}$$

$$\text{Radial strain } \varepsilon_r = \frac{\delta_r + \frac{\partial \delta_r}{\partial r}dr - \delta_r}{dr} = \frac{\partial \delta_r}{\partial r} = \frac{(\sigma_r - \nu\,\sigma_\theta)}{E}$$

$$\text{Force balance} = (\sigma_r + d\sigma_r)(r+dr)\,d\theta\,dz - \sigma_r\,r\,d\theta\,dz - 2\,\sigma_\theta \sin\left(\frac{d\theta}{2}\right)dr\,dz = 0$$

$$(\sigma_r + d\sigma_r)(r + dr)\, d\theta\, dz - \sigma_r\, r\, d\theta\, dz - 2\,\sigma_\theta \sin\left(\frac{d\theta}{2}\right) dr\, dz = 0$$

$$rearranging\; r\frac{d\sigma_r}{dr}\, d\theta\, dz + \sigma_r\, d\theta\, dz - \sigma_\theta\, d\theta\, dz = 0$$

$$or\;\; \sigma_\theta = \sigma_r + r\frac{d\sigma_r}{dr}$$

$$\frac{\delta_r}{r} = \frac{(\sigma_\theta - \nu\,\sigma_r)}{E}$$

$$\frac{\partial\delta_r}{\partial r} = \frac{(\sigma_r - \nu\,\sigma_\theta)}{E}$$

$$\frac{\delta_r}{r} = \frac{\left(\sigma_r + r\dfrac{d\sigma_r}{dr} - \nu\,\sigma_r\right)}{E}$$

$$\frac{\partial\delta_r}{\partial r} = \frac{\left(\sigma_r - \nu\,\sigma_r - \nu\, r\dfrac{d\sigma_r}{dr}\right)}{E}$$

$$\frac{\partial\delta_r}{\partial r} = \frac{1}{E}\left(\sigma_r + r\frac{d\sigma_r}{dr} - \nu\sigma_r\right) + \frac{r}{E}\begin{pmatrix} r\dfrac{d^2\sigma_r}{dr^2} + \\ 2\dfrac{d\sigma_r}{dr} - \nu\dfrac{d\sigma_r}{dr} \end{pmatrix}$$

$$3r\frac{d\sigma_r}{dr} + r^2\frac{d^2\sigma_r}{dr^2} = 0$$

$$\frac{d\sigma_r}{dr} + \frac{d^2(r\,\sigma_r)}{dr^2} = 0$$

$$\frac{d\sigma_r}{dr} + \frac{d^2(r\,\sigma_r)}{dr^2} = 0$$

$$\sigma_r + \frac{d(r\,\sigma_r)}{dr} + C_1 = 0$$

$$\frac{d(r^2\,\sigma_r)}{dr} + C_1\,r = 0$$

$$r^2\,\sigma_r + C_1\frac{r^2}{2} + C_2 = 0$$

$$\sigma_r + \frac{C_1}{2} + \frac{C_2}{r^2} = 0$$

Two conditions are required to express radial stress in terms of radius.

$$\sigma_r = -p_i \quad at\ r = r_i$$

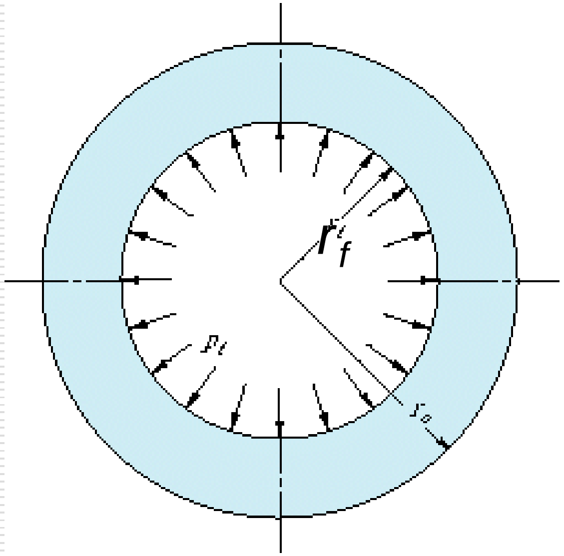
$$\sigma_r = -p_o \quad at\ r = r_o$$

$$\frac{C_1}{2} + \frac{C_2}{r_i^2} = p_i$$

$$\frac{C_1}{2} + \frac{C_2}{r_o^2} = p_o$$

Radial stress $\sigma_r = \dfrac{p_i r_i^2 - p_o r_o^2 + \left(r_i r_o / r\right)^2 \left(p_o - p_i\right)}{r_o^2 - r_i^2}$

Circumferential stress $\sigma_\theta = \dfrac{p_i r_i^2 - p_o r_o^2 - \left(r_i r_o / r\right)^2 \left(p_o - p_i\right)}{r_o^2 - r_i^2}$



Finding Stress in Hub

Circumferential stress $\sigma_\theta = \dfrac{p_f\, r_f^2 \left(1 + \left(r_o / r\right)^2\right)}{r_o^2 - r_f^2}$

Radial stress $\sigma_r = \dfrac{p_f\, r_f^2 \left(1 - \left(r_o / r\right)^2\right)}{r_o^2 - r_f^2}$

$$\Longrightarrow \boxed{\sigma_{\theta,\max} = \frac{p_f \left(r_f^2 + r_o^2\right)}{r_o^2 - r_f^2} \qquad \sigma_{r,\max} = -p_f}$$

Circumferential strain $\varepsilon_\theta = \dfrac{\delta_{rh}}{r_f} = \dfrac{\left(\sigma_\theta - \nu_h\, \sigma_r\right)}{E}$

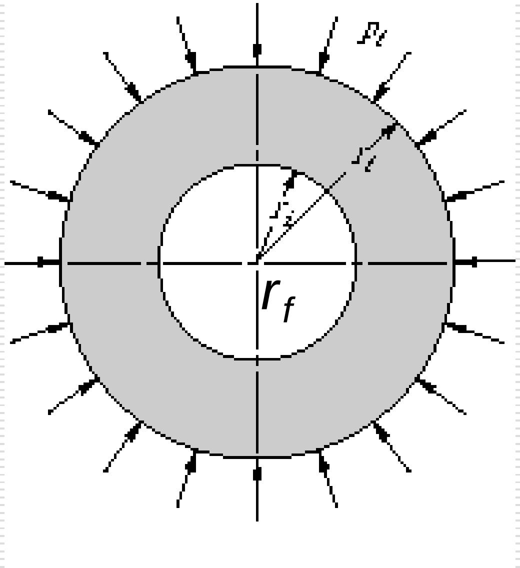
$$\Longrightarrow \boxed{\frac{\delta_{r_h}}{r_f} = \frac{p_f}{E}\left(\frac{r_f^2 + r_o^2}{r_o^2 - r_f^2} + \nu_h\right)}$$

Finding Stress in shaft

Circumferential stress $\sigma_\theta = -p_f\, r_f^2 \left(\dfrac{1+(r_i/r)^2}{r_f^2 - r_i^2} \right)$

Radial stress $\sigma_r = -p_f r_f^2 \left(\dfrac{1-(r_i/r)^2}{r_f^2 - r_i^2} \right)$

$$\boxed{\begin{aligned} \sigma_{\theta,\max} &= -p_f\, r_f^2 \left(\frac{2}{r_f^2 - r_i^2} \right) \\ \sigma_{r,\max} &= -p_f \end{aligned}}$$



Circumferential strain $\varepsilon_\theta = \dfrac{\delta_{rs}}{r_f} = \dfrac{(\sigma_\theta - \nu_s\, \sigma_r)}{E}$

$$\boxed{\frac{\delta_{r_s}}{r_f} = -\frac{p_f}{E_s} \left(\frac{r_i^2 + r_f^2}{r_f^2 - r_i^2} - \nu_s \right)}$$

Total interference $\delta_{\mathrm{r}} = \delta_{rh} - \delta_{rs}$

or $\delta_{\mathrm{r}} = r_f p_f \left[\frac{r_o^2 + r_f^2}{E_h\left(r_o^2 - r_f^2\right)} + \frac{\nu_h}{E_h} + \frac{r_i^2 + r_f^2}{E_s\left(r_f^2 - r_i^2\right)} - \frac{\nu_s}{E_s} \right]$

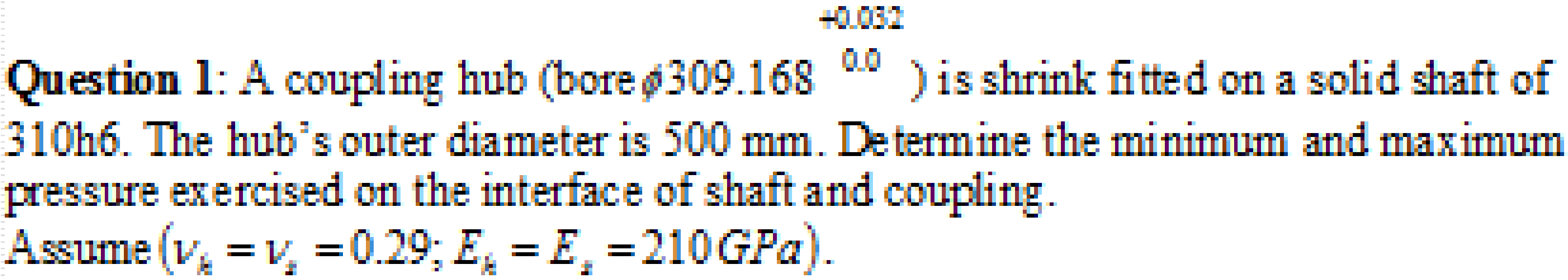
Ex: A wheel hub is press fitted on a 105 mm diameter solid shaft. The hub and shaft material is AISI 1080 steel (E = 207 GPa). The hub's outer diameter is 160mm. The radial interference between shaft and hub is 65 microns. Determine the pressure exercised on the interface of shaft and wheel hub.

If hub and shaft are made of same materials: $\delta_{\mathrm{r}} = \frac{r_f p_f}{E} \left[\frac{r_o^2 + r_f^2}{\left(r_o^2 - r_f^2\right)} + \frac{r_i^2 + r_f^2}{\left(r_f^2 - r_i^2\right)} \right]$

If shaft is solid: $\delta_{\mathrm{r}} = \frac{r_f p_f}{E} \left[\frac{2 r_o^2}{\left(r_o^2 - r_f^2\right)} \right]$

ANS: $p_f = 73$ MPa

Iterations !!!!



Question 1: A coupling hub (bore $\phi 309.168^{+0.032}_{0.0}$) is shrink fitted on a solid shaft of 310h6. The hub's outer diameter is 500 mm. Determine the minimum and maximum pressure exercised on the interface of shaft and coupling.
Assume $(\nu_h = \nu_s = 0.29; E_h = E_s = 210\,GPa)$.

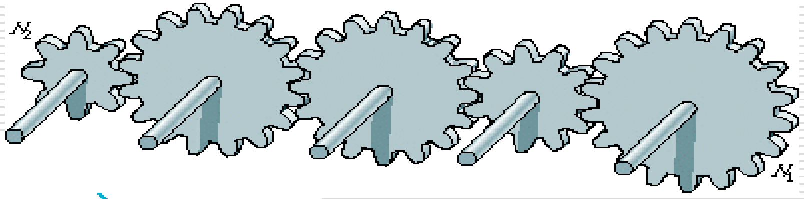
Through interference fit torque can be transmitted, which can be estimated with a simple friction analysis at the interface.

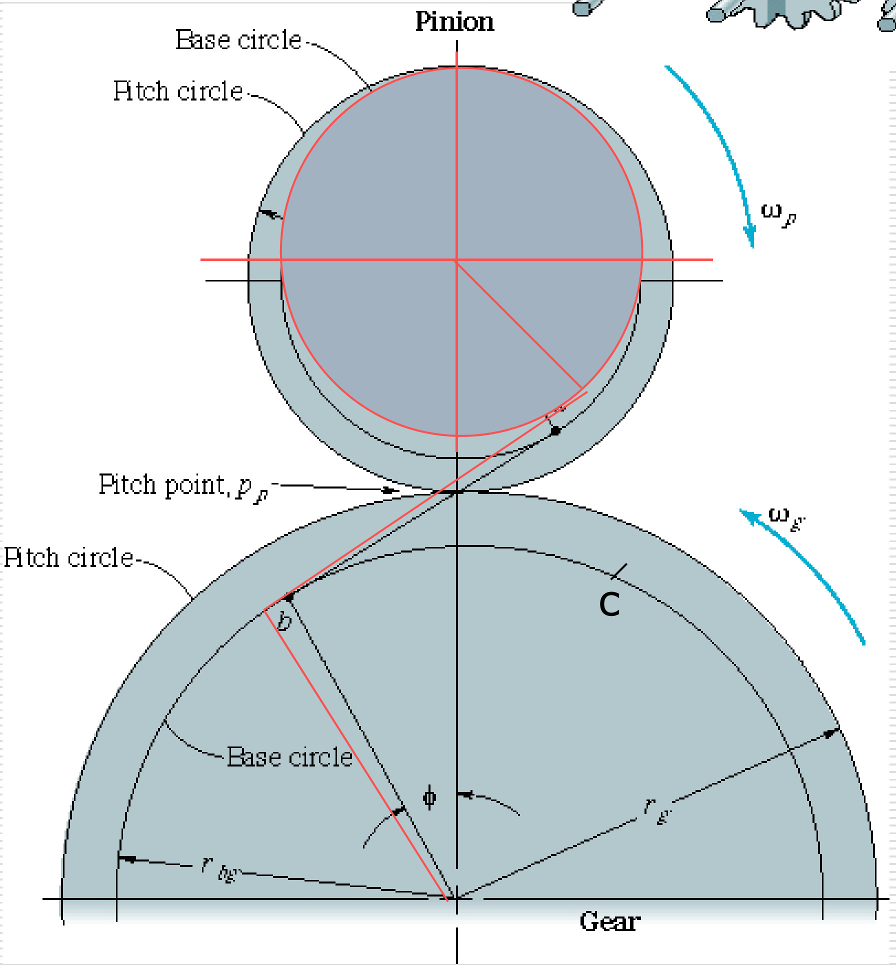
$$F_f = \mu N = \mu(p_f\, A)$$

$$F_f = \mu(p_f\, \pi\, d_f\, L)$$

$$Torque\; T = \frac{\pi}{2}\mu\, p_f\, d^2\, L$$

GEARS





Pressure angle ??

$$\phi_I = \cos^{-1}\left(\frac{R_{bg}}{R_g}\right)$$

Tooth curves of the mating
Teeth need to be tangent
to each other.

Line of action is tangent to
Both pinion & gear base
Circles.

On changing center distance
Line of action still remains
Tangent to both base circles
But slope changes.

Design of Spur Gear

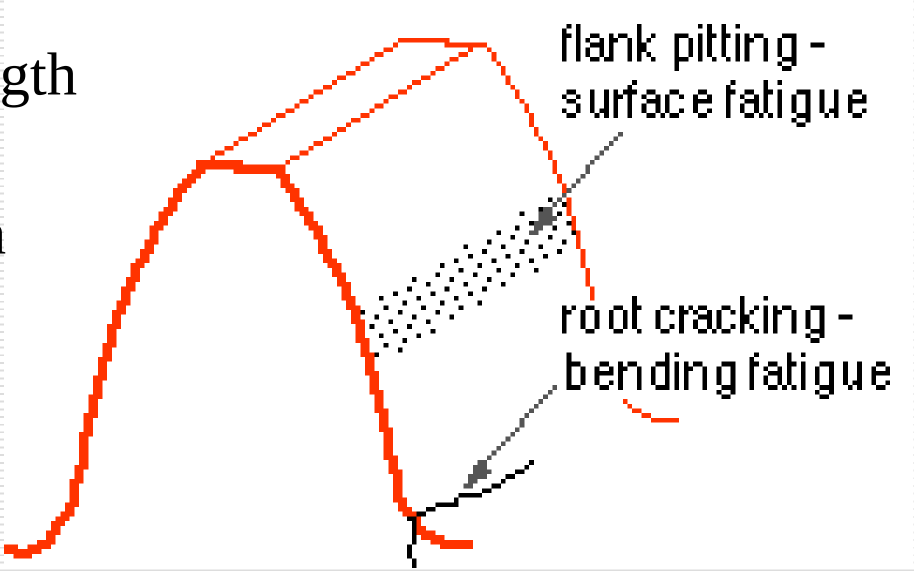
$$\sigma_{b,\text{static}} \le \text{Elastic strength}$$

$$\sigma_{b,\text{dynamic}} \le \text{Bending endurance strength}$$

$$\sigma_{contact} \le \text{Surface endurance strength}$$

Core

Case



Hardness; Strength > 1 GPa

AGMA introduced velocity factor in terms of pitch line velocity (m/s) in Lewis equation.

$$K_v = \frac{3.05 + V}{3.05} \quad (\mathit{cast\ iron, cast\ profile})$$

$$K_v = \frac{6.01 + V}{6.01} \quad (\mathit{Cut\ or\ milled\ profile})$$

$$K_v = \frac{3.56 + \sqrt{V}}{3.56} \quad (\mathit{Hobbed\ or\ shaped\ profile})$$

$$K_v = \sqrt{\frac{5.56 + \sqrt{V}}{5.56}} \quad (\mathit{Shaved\ or\ ground\ profile})$$

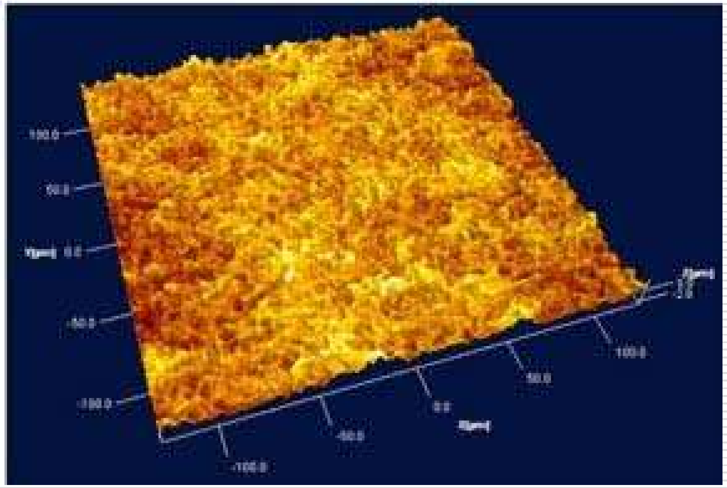
$AGMA$ Lewis Eq.

$$\sigma_b = \frac{K_v\, W_t}{F\, Y\, m}$$

Useful for preliminary estimation of gear size.

For V = 15 m/s , K_v

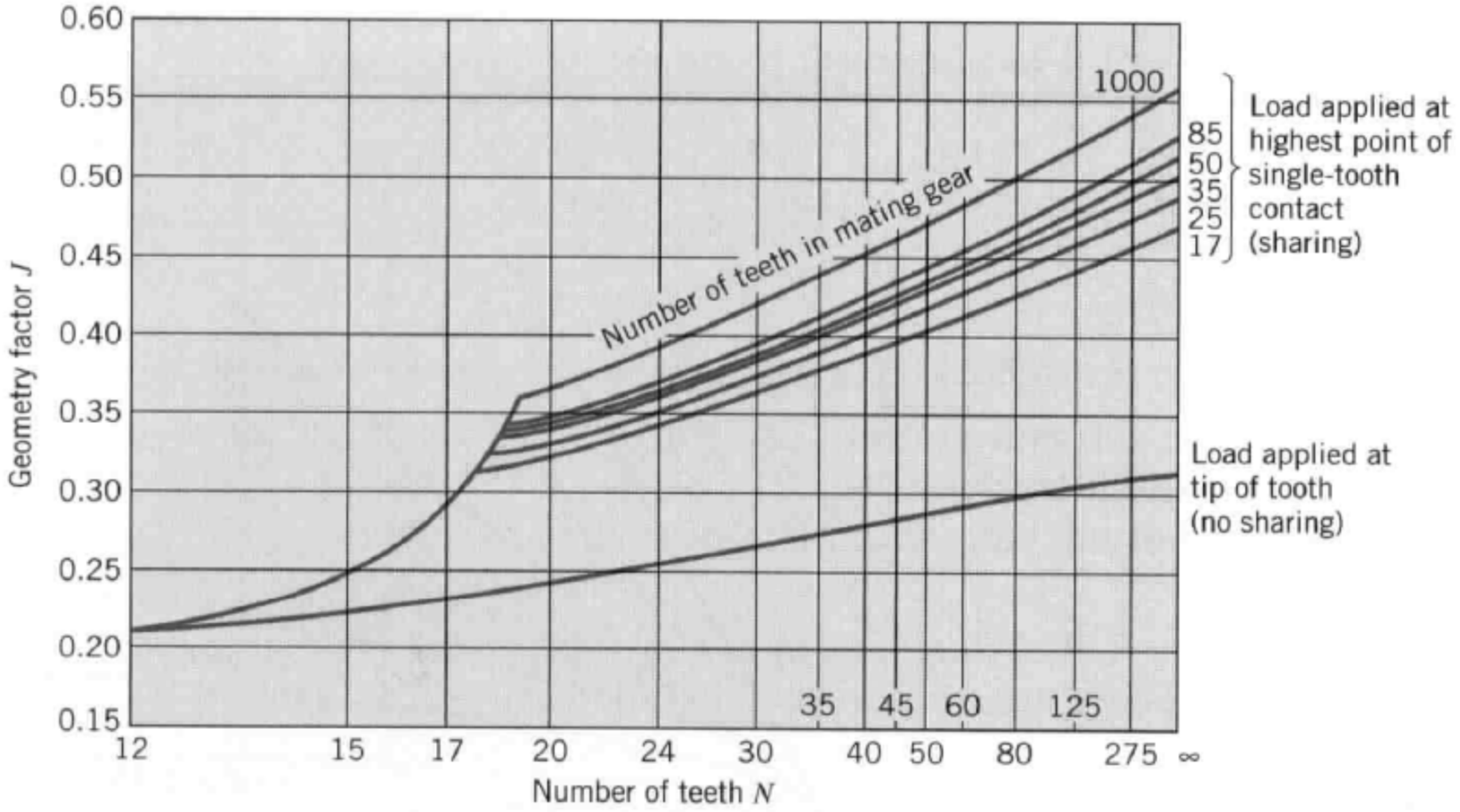
- Cast iron, cast profile = 5.2
- Cut or milled profile = 3.5
- Hobbed or shaped profile = 2.1
- Shaved or ground = 1.3



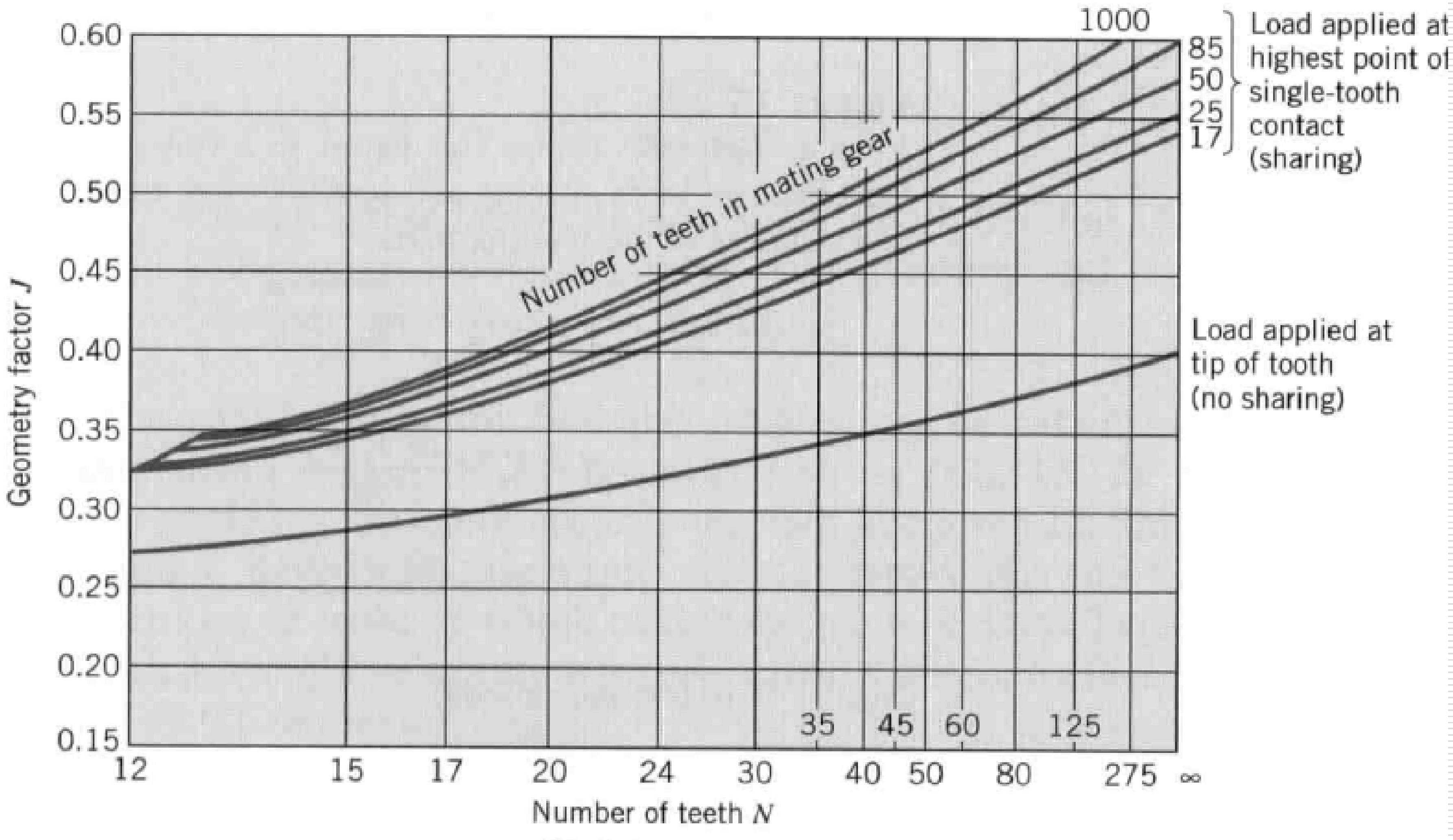
AGMA Bending Stress Equation

$$\sigma_b = \frac{K_v W_t}{F\, m\, J} K_a K_B K_m$$

$J = AGMA\ bending\ Geometry\ Factor$ depends on pressure angle, point of loading

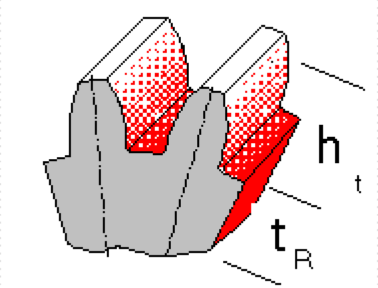


(b) 20° full-depth teeth



25° full-depth teeth

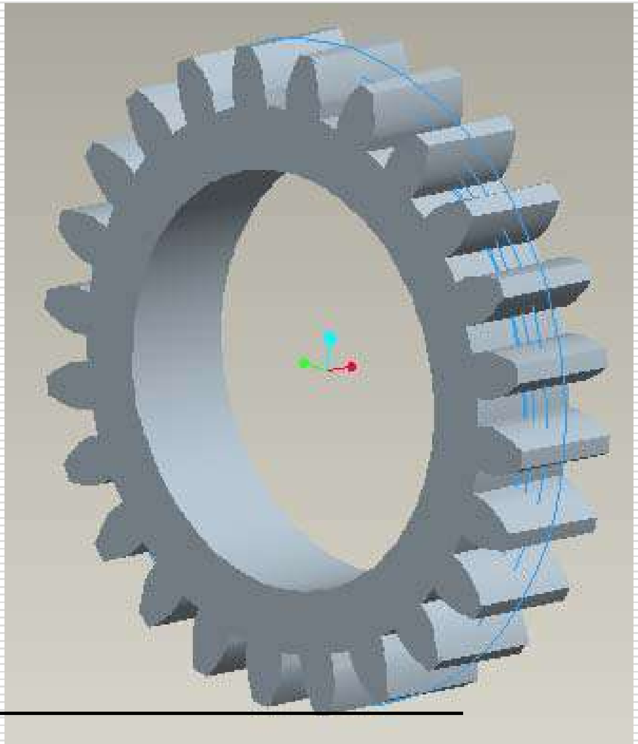
$$\sigma_b = \frac{K_v W_t}{F\, m\, J} K_a K_B K_m$$



Rim thickness factor

$$K_B = -2\, m_B + 3.4 \qquad 0.5 \le \mathrm{m}_{\mathrm{B}} < 1.2$$

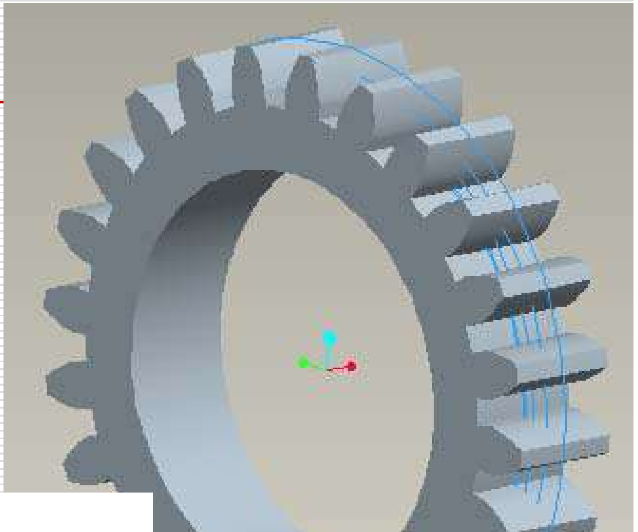
$$K_B = 1.0 \qquad m_B \ge 1.2 \quad where \qquad m_B = \frac{t_R}{h_t}$$

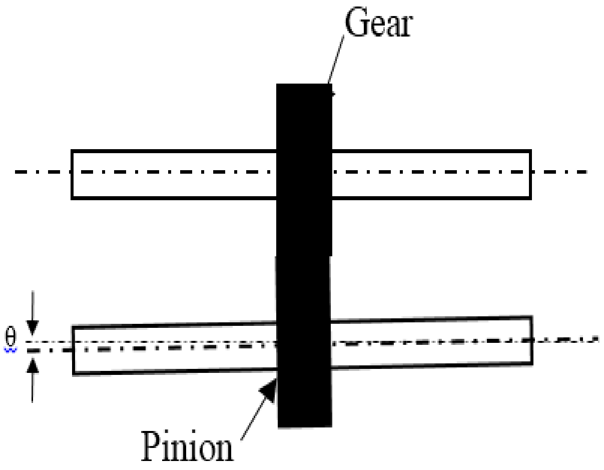


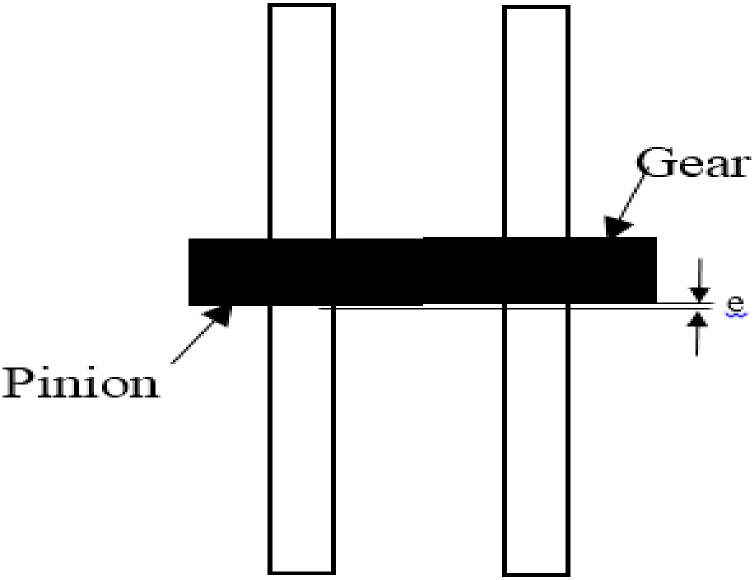
	Driven Machines			
Power Source	**Uniform**	**Light shock**	**Moderate shock**	**Heavy shock**
			Application factor, K_a	
Uniform (Electric motor, turbine)	1.00	1.25	1.50	1.75
Light shock (Multicylinder)	1.20	1.40	1.75	2.25
Moderate shock	1.30	1.70	2.00	2.75

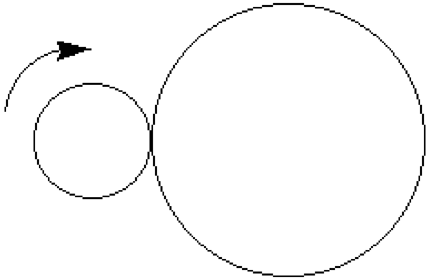
Load distribution factor K_m	
Face width, mm	K_m
< 50	1.6
150	1.7
250	1.8
>500	2.0

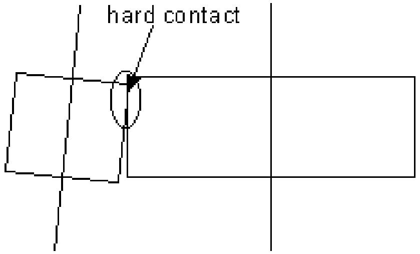
$$\sigma_b = \frac{K_v W_t}{F\, m\, J} K_a K_B K_m$$









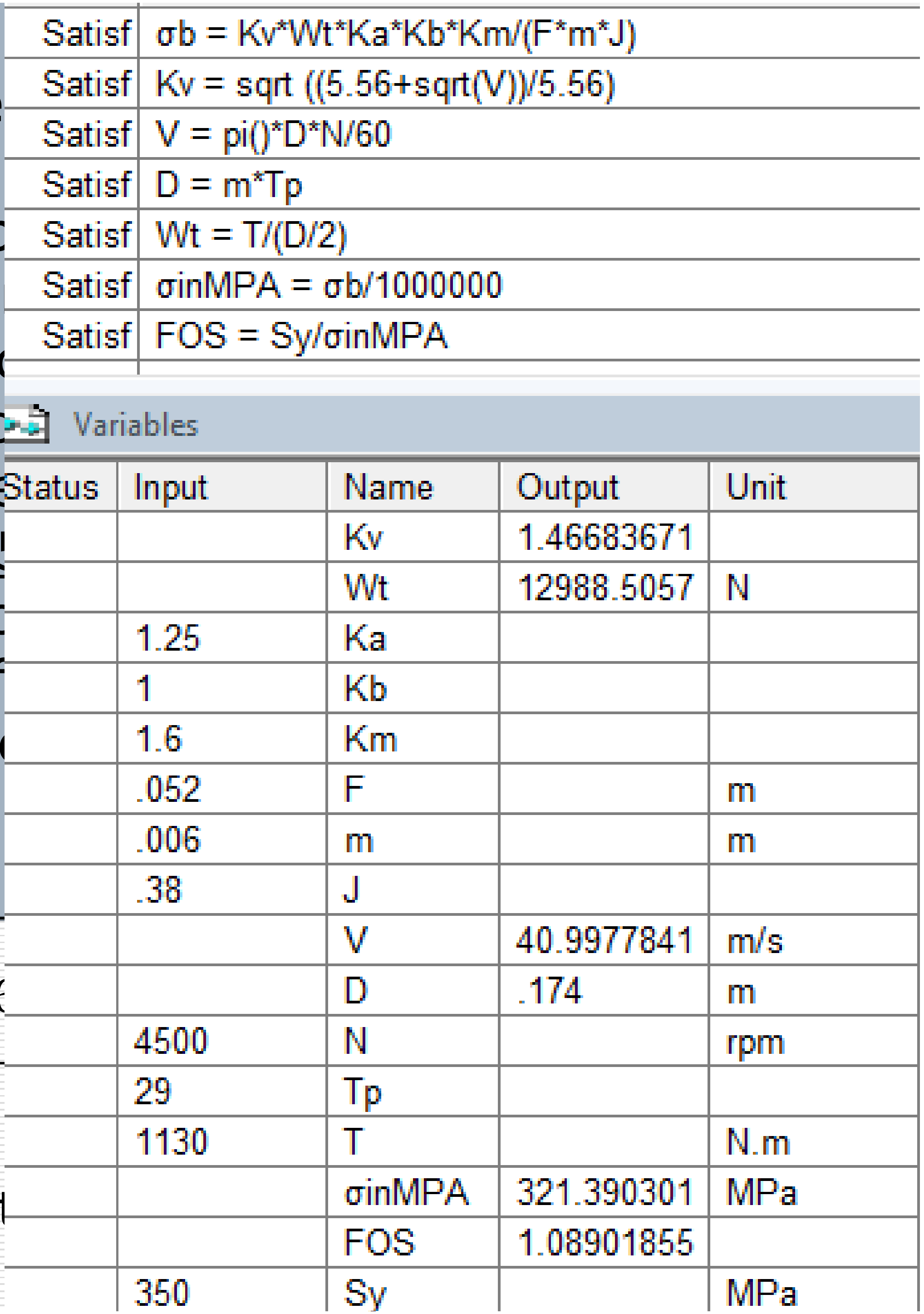


Case Study: Determine safe
grounded) pinion of spur gea
1130Nm @ 4500 rpm with sp
the pressure angle = 20°, nu
module(m)=6mm. The corre
material varies in the range o
Standard deviations in torque
rpm and in pitch diameter 0.
manufacturing errors, the fac
52mm. Due to possibilities of
effective face width may vari
52mm.

$$K_v = \sqrt{\frac{5.56+\sqrt{V}}{5.56}}$$

$$K_v = \sqrt{\frac{5.5}{\ }}$$

Factor J for 29 number of teet



Satisf	σb = Kv*Wt*Ka*Kb*Km/(F*m*J)
Satisf	Kv = sqrt ((5.56+sqrt(V))/5.56)
Satisf	V = pi()*D*N/60
Satisf	D = m*Tp
Satisf	Wt = T/(D/2)
Satisf	σinMPA = σb/1000000
Satisf	FOS = Sy/σinMPA

Variables

Status	Input	Name	Output	Unit
		Kv	1.46683671	
		Wt	12988.5057	N
	1.25	Ka		
	1	Kb		
	1.6	Km		
	.052	F		m
	.006	m		m
	.38	J		
		V	40.9977841	m/s
		D	.174	m
	4500	N		rpm
	29	Tp		
	1130	T		N.m
		σinMPA	321.390301	MPa
		FOS	1.08901855	
	350	Sy		MPa

Statistical Approach

$$\sigma_b = f\left(K_v, W_t, F, m, J, K_a, K_m, K_B\right)$$

$$\sigma_{\sigma_b} = \sqrt{\begin{array}{l}\left(\frac{\partial \sigma_b}{\partial K_v}\right)^2 \sigma_{K_v}^2 + \left(\frac{\partial \sigma_b}{\partial W_t}\right)^2 \sigma_{W_t}^2 + \left(\frac{\partial \sigma_b}{\partial F}\right)^2 \sigma_F^2 + \left(\frac{\partial \sigma_b}{\partial m}\right)^2 \sigma_m^2 + \\ \left(\frac{\partial \sigma_b}{\partial J}\right)^2 \sigma_J{}^2 + \left(\frac{\partial \sigma_b}{\partial K_a}\right)^2 \sigma_{K_a}^2 + \left(\frac{\partial \sigma_b}{\partial K_m}\right)^2 \sigma_{K_m}^2 + \left(\frac{\partial \sigma_b}{\partial K_B}\right)^2 \sigma_{K_B}^2\end{array}}$$

W_t depends on the applied torque (T) and pitch diameter (D) (W_t=2T/D). Therefore *f*(W_t) is to be replaced by *f(T,D).*

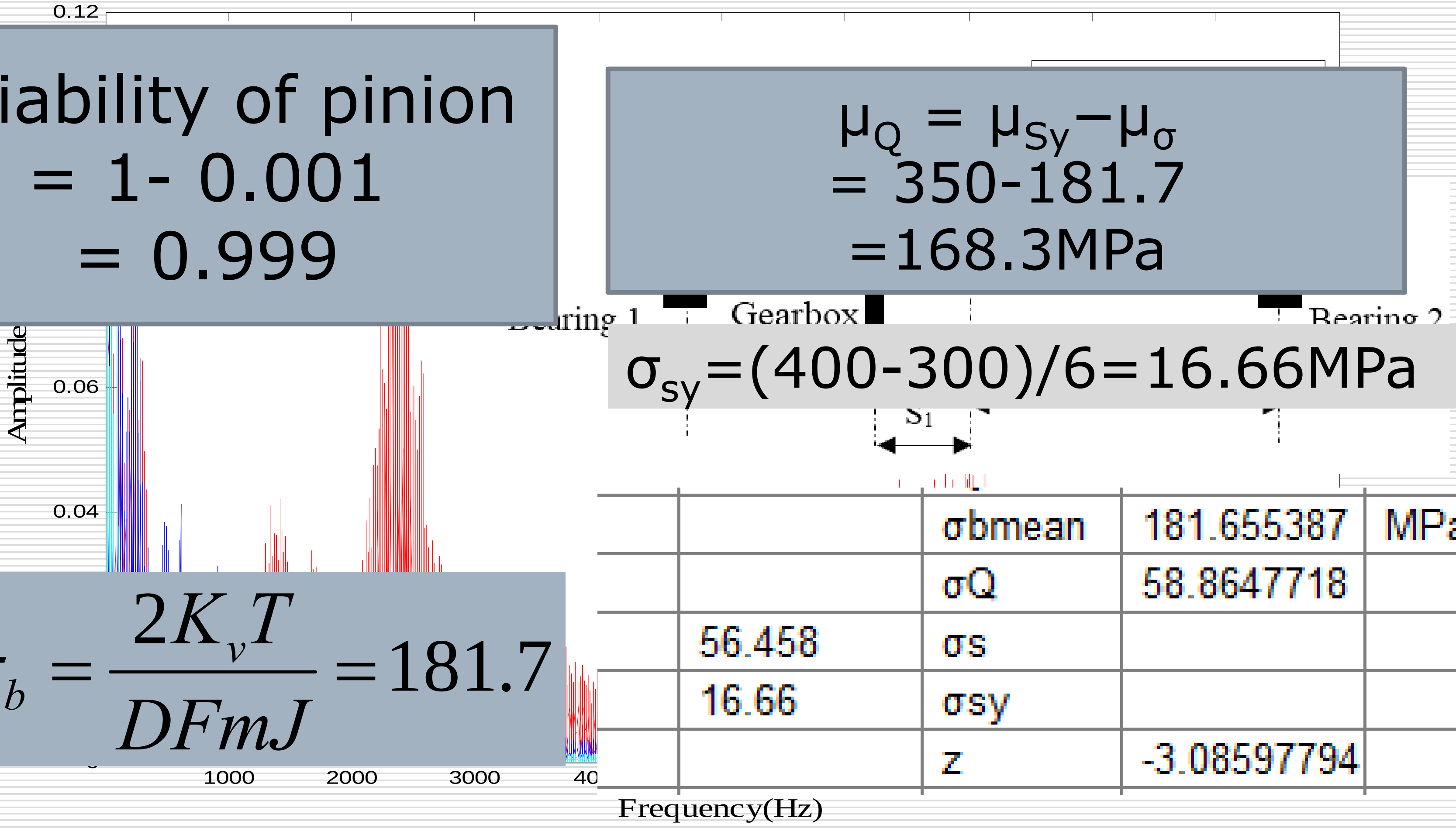
K_v is function pitch line velocity (V). The V is function of angular speed (N) and pitch diameter (D).

$$\sigma_{\sigma_b} = \sqrt{\begin{aligned}&\left(\frac{\partial \sigma_b}{\partial T}\right)^2 \sigma_T^2 + \left(\frac{\partial \sigma_b}{\partial F}\right)^2 \sigma_F^2 + \left(\frac{\partial \sigma_b}{\partial m}\right)^2 \sigma_m^2 + \left(\frac{\partial \sigma_b}{\partial J}\right)^2 \sigma_J^{\,2} + \left(\frac{\partial \sigma_b}{\partial K_a}\right)^2 \sigma_{K_a}^2 \\ &+ \left(\frac{\partial \sigma_b}{\partial K_m}\right)^2 \sigma_{K_m}^2 + \left(\frac{\partial \sigma_b}{\partial K_B}\right)^2 \sigma_{K_B}^2 + \left(\frac{\partial \sigma_b}{\partial D}\right)^2 \sigma_D^2 + \left(\frac{\partial \sigma_b}{\partial N}\right)^2 \sigma_N^2\end{aligned}}$$

Is there any to consider the over load factor (K_a)?

AGMA bending geometry factor (J) is function of number of teeth and nominal pressure angle, which will have zero standard deviation. No need to consider standard deviation of J.
Similarly, deviation in value of module (m) is almost negligible.

$$\sigma_{\sigma_b} = \sqrt{\left(\frac{\partial \sigma_b}{\partial T}\right)^2 \sigma_T^2 + \left(\frac{\partial \sigma_b}{\partial F}\right)^2 \sigma_F^2 + \left(\frac{\partial \sigma_b}{\partial K_m}\right)^2 \sigma_{K_m}^2 + \left(\frac{\partial \sigma_b}{\partial K_B}\right)^2 \sigma_{K_B}^2 + \left(\frac{\partial \sigma_b}{\partial D}\right)^2 \sigma_D^2 + \left(\frac{\partial \sigma_b}{\partial N}\right)^2 \sigma_N^2}$$



Reliability of pinion
= 1- 0.001
= 0.999

$$\mu_Q = \mu_{Sy} - \mu_\sigma$$
$$= 350-181.7$$
$$= 168.3 \text{MPa}$$

$$\sigma_{sy} = (400-300)/6 = 16.66 \text{MPa}$$

$$\sigma_b = \frac{2K_vT}{DFmJ} = 181.7$$

		σbmean	181.655387	MPa
		σQ	58.8647718	
	56.458	σs		
	16.66	σsy		
		z	-3.08597794	

$$\sigma_{\sigma_b} = \sqrt{\left(\frac{\partial \sigma_b}{\partial T}\right)^2 \sigma_T^2 + \left(\frac{\partial \sigma_b}{\partial F}\right)^2 \sigma_F^2 + \left(\frac{\partial \sigma_b}{\partial D}\right)^2 \sigma_D^2 + \left(\frac{\partial \sigma_b}{\partial N}\right)^2 \sigma_N^2} = 56.458\,MPa$$

Ex: A gear pair (Z_P=23, ϕ=20°, Z_g =24, m=1.75, F=10.0 mm) transmits 8 N.m torque from crankshaft (rotational speed 8000 rpm) of single cylinder IC engine to wheels. Bore diameter of pinion is 17 mm, and bore dia of gear is 20 mm. Use AGMA bending stress formula to determine the maximum bending stress. Assume gears are grounded.

Given: F = 10 mm, m = 1.75, W_t = 8000/(23*1.75*0.5)

$$\sigma_b = \frac{K_v W_t}{F\, m\, J} K_a K_B K_m$$

	Driven Machines			
Power Source	**Uniform**	**Light shock**	**Moderate shock**	**Heavy shock**
			Application factor, K_a	
Uniform (Electric motor, turbine)	1.00	1.25	1.50	1.75
Light shock (Multicylinder)	1.20	1.40	1.75	2.25
Moderate shock	1.30	1.70	2.00	2.75

Load distribution factor K_m	
Face width, mm	K_m
< 50	1.6

$$K_a = 2.0 \qquad K_m = 1.6$$

$$\boxed{K_v = \sqrt{\frac{5.56+\sqrt{V}}{5.56}} \quad (ground\ gears)}$$

$$d_p = 23*1.75 = 40.25\,\text{mm}$$

$$V = \frac{\pi\ d_p\ N}{60} = \frac{\pi\,(40.25)\,8000}{60} \rightarrow 16.86\, m/s$$

$$K_v = \sqrt{\frac{5.56+\sqrt{V}}{5.56}} = 1.3185$$

$$d_{proot} = d_p - 2*1.25*1.75 = 35.875$$

$$h_t = 2.25*1.75 = 3.9375\, mm$$

$$t_R = 0.5\left(d_{proot} - Bore_p\right) = 9.4375$$

$$m_B > 1.2 \Rightarrow K_B = 1$$

$$\boxed{\sigma_b = \frac{K_v W_t}{F\, m\, J} K_a K_B K_m}$$